Modello risorse-consumatori di Rosenzweig-MacArthur prede-predatore

x_1 = densità/biomassa di risorse/prede

x_2 = " consumatori/predatori

$$\dot{x}_1 = r x_1\left(1-\frac{x_1}{k}\right) - \frac{a x_1}{b+x_1} x_2$$

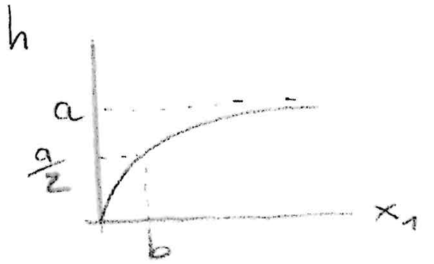
crescita logistica

mortalità per predazione

$h(x_1) = \frac{a x_1}{b+x_1}$ → Risposta funzionale di Holling di tipo II

r = tasso intrinseco di crescita

k = capacità portante



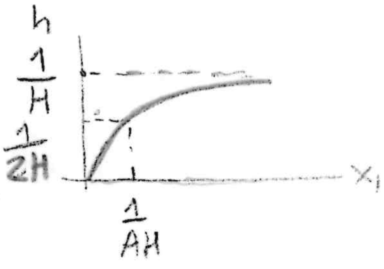
a = massima capacità predatoria (è pari al max di h)

b = costante di semi-saturazione (è il valore di x_1 per il quale h è metà del suo valore massimo)

<u>NOTA</u>: Altra formulazione per $h(x_1) = \frac{A x_1}{1+AHx_1}$

A = attack rate

H = handling time



$$\Rightarrow \quad a = \frac{1}{H} \qquad b = \frac{1}{AH} \quad \rightarrow \quad A = \frac{a}{b} \qquad H = \frac{1}{a}$$

$$\dot{x}_2 = e\,\frac{a x_1}{b+x_1}\,x_2 - m x_2$$

crescita per predazione

mortalità naturale

m = tasso di mortalità

e = efficienza di conversione

$$\boxed{\begin{aligned}\dot{x}_1 &= r x_1\left(1-\frac{x_1}{k}\right) - a\,\frac{x_1}{b+x_1}\,x_2\\ \dot{x}_2 &= e\,\frac{a x_1}{b+x_1}\,x_2 - m x_2\end{aligned}}$$

<u>Isocline - equilibri - traiettorie - stabilità</u>

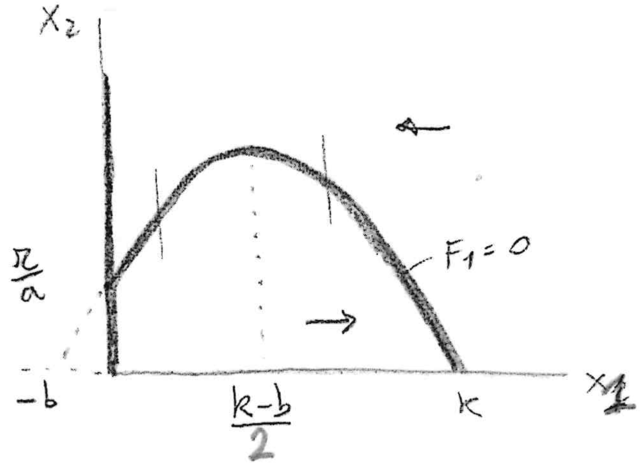
$$\dot{x}_1 = x_1\left[r\left(1-\frac{x_1}{k}\right) - \frac{a x_2}{b+x_1}\right] = x_1 \cdot F_1(x_1, x_2)$$

$$\dot{x}_2 = x_2\left[e\,\frac{a x_1}{b+x_1} - m\right] = x_2 \cdot F_2(x_1)$$

<u>Isocline</u> $\dot{x}_1 = 0$ <

$x_1 = 0$

$x_2 = \frac{r}{a}\left(1-\frac{x_1}{k}\right)(b+x_1) \rightsquigarrow F_1(x_1,x_2)=0$



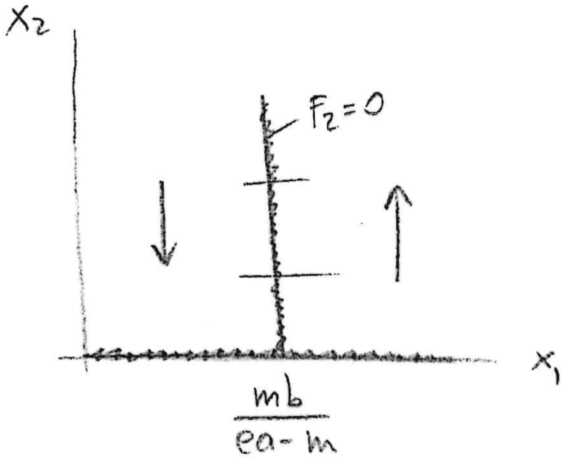
$(k>b)$

Isocline $\dot{x}_2 = 0$ — $x_2 = 0$

— $x_1 = \dfrac{mb}{ea-m}$ $\Rightarrow F_2(x_1)=0$ $ea-m>0$

$\downarrow$

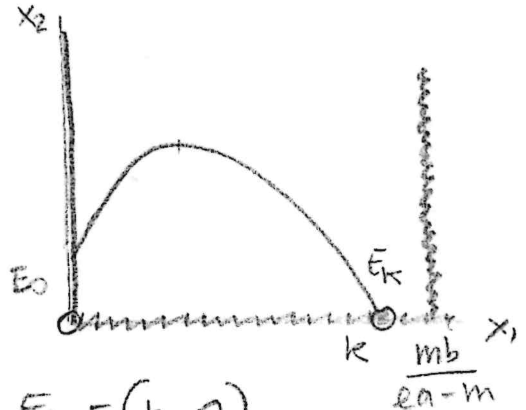
Se la risorsa x_1 è infinita, il consumatore è in grado di crescere ($\dot{x}_2 > 0$)



Equilibri $\longrightarrow$ $\cap$ isocline $\dot{x}_1 = 0$ e $\dot{x}_2 = 0$

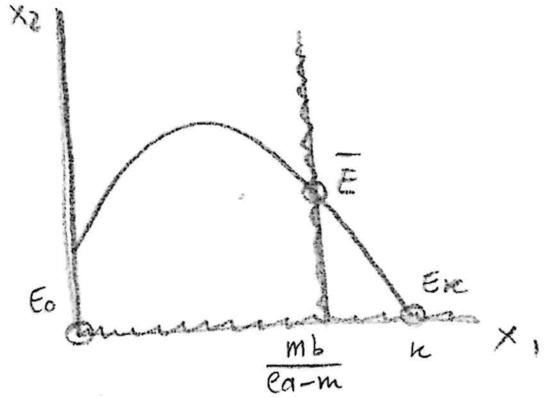
$\hookrightarrow x_i \geq 0$

- $k < \dfrac{mb}{ea-m}$



$E_0 = (0, 0)$ $E_k = (k, 0)$

- $k > \dfrac{mb}{ea-m}$



$E_0 = (0,0)$ $E_k = (k, 0)$ $\bar{E} \,/\, \begin{cases} F_1 = 0 \\ F_2 = 0 \end{cases}$ $\bar{x}_1 = \dfrac{mb}{ea-m}$

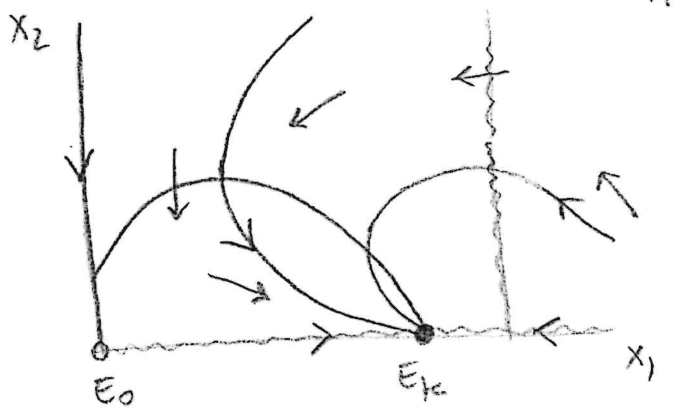
NOTA : $\dfrac{mb}{ea-m} > \dfrac{k-b}{2} \Rightarrow \bar{E}$ è a dx del max della parabola

$\dfrac{mb}{ea-m} < \dfrac{k-b}{2} \Rightarrow \bar{E}$ è a sx " "

NOTA : Gli assi sono invarianti! $x_i(0)=0 \rightarrow \dot{x}_i(0)=0 \rightarrow x_i(t)=0$

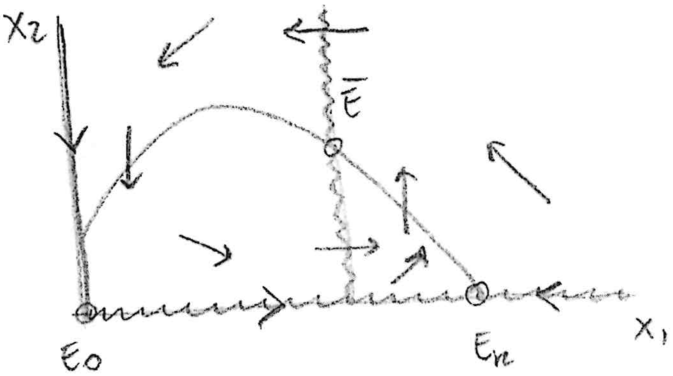
Traiettorie

$k < \dfrac{mb}{ea-m}$



E_0 INST
E_k STAB

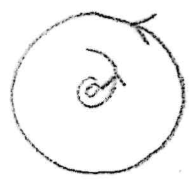
$k > \dfrac{mb}{ea-m}$



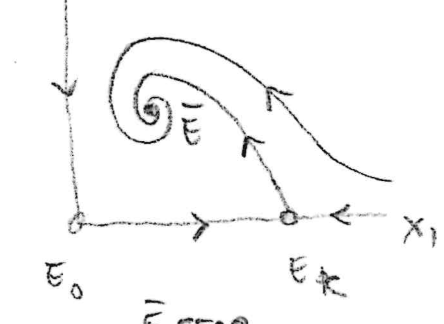
E_0, E_k INST

Nell'intorno di $\bar{E}$

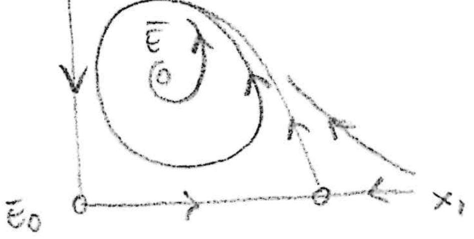
oppure



$\Downarrow$ $\Downarrow$



$\bar{E}$ STAB



$\bar{E}$ INST

Stabilità

$$\dot{x}_1 = r x_1 \left(1 - \frac{x_1}{k}\right) - \frac{a x_1}{b + x_1} x_2 = x_1 \left[r\left(1 - \frac{x_1}{k}\right) - \frac{a}{b+x_1} x_2 \right] = x_1 F_1(x_1, x_2) = f_1$$

$$\dot{x}_2 = e \frac{a x_1}{b + x_1} x_2 - m x_2 = x_2 \left[\frac{e a x_1}{b + x_1} - m \right] = x_2 F_2(x_1) = f_2$$

All'equilibrio: $\begin{cases} \dot{x}_1 = 0 \\ \dot{x}_2 = 0 \end{cases} \rightarrow \begin{cases} x_1 F_1 = 0 \\ x_2 F_2 = 0 \end{cases}$ da cui si hanno i tre equilib.

$\begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases} \rightarrow E_0 = (0, 0)$ | $\begin{cases} x_2 = 0 \\ F_1 = 0 \end{cases} \rightarrow \begin{matrix} x_2 = 0 \\ x_1 = k \end{matrix} \rightarrow E_k = (k, 0)$

$\begin{cases} F_1 = 0 \\ F_2 = 0 \end{cases} \rightarrow \bar{E} = (\bar{x}_1, \bar{x}_2) \rightsquigarrow \in$ I quadrante se $0 < \frac{mb}{ea - m} < k$
(vedi isocline)

Stabilità via linearizzazione

$$J = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{vmatrix} = \begin{vmatrix} F_1 + x_1 \frac{\partial F_1}{\partial x_1} & x_1 \frac{\partial F_1}{\partial x_2} \\ x_2 \frac{\partial F_2}{\partial x_1} & F_2 + x_2 \underbrace{\frac{\partial F_2}{\partial x_2}}_{0} \end{vmatrix}$$

- Stabilità di $E_0 = (0, 0)$

$$J_{E_0} = \begin{vmatrix} F_1(0,0) & 0 \\ 0 & F_2(0) \end{vmatrix} = \begin{vmatrix} r & 0 \\ 0 & -m \end{vmatrix}$$

$\lambda_1 = r > 0$
$\lambda_2 = -m < 0$

E_0 INSTABILE
SELLA

- Stabilità di $E_k = (k, 0)$ $\overset{\text{NOTA}}{\rightsquigarrow} F_1(k, 0) = 0$ $\frac{\partial F_1}{\partial x_1} = -\frac{r}{k} + \frac{a x_2}{(b + x_1)^2}$

$$J_{E_k} = \begin{vmatrix} k\left(-\frac{r}{k}\right) & * \\ 0 & \frac{eak}{b+k} - m \end{vmatrix} = \begin{vmatrix} -r & * \\ 0 & \frac{eak}{b+k} - m \end{vmatrix}$$

$\lambda_1 = -r < 0$

$\lambda_2 = \frac{eak}{b+k} - m \rightsquigarrow$ il segno di λ_2 determina la stabilità di E_k

$\Rightarrow \lambda_2 < 0 \qquad \frac{eak}{b+k} - m < 0 \rightarrow eak - mb - mk < 0$

$\frac{mb}{ea-m} > k \qquad E_k$ è asint. stab.

$\Rightarrow \lambda_2 > 0 \quad ---\ \frac{mb}{ea-m} < k \qquad E_k$ è instabile $\rightsquigarrow$ sella

- Stabilità di $\bar{E} \overset{\text{NOTA}}{\rightsquigarrow} F_1(\bar{x}_1, \bar{x}_2) = 0$ e $F_2(\bar{x}_1) = 0$ con $\bar{x}_1 > 0$ e $\bar{x}_2 > 0$

$$J_{\bar{E}} = \begin{vmatrix} \bar{x}_1 \frac{\partial F_1}{\partial x_1} & \bar{x}_1 \frac{\partial F_1}{\partial x_2} \\ \bar{x}_2 \frac{\partial F_2}{\partial x_1} & 0 \end{vmatrix}$$

$\frac{\partial F_1}{\partial x_1} = -\frac{r}{k} + \frac{a x_2}{(b+x_1)^2} \quad \Big| \quad \frac{\partial F_1}{\partial x_2} = -\frac{a}{b+x_1} \quad \Big| \quad \frac{\partial F_2}{\partial x_1} = \frac{eab}{(b+x_1)^2}$

$$\text{tr}\, J_{\bar{E}} = \left[-\frac{r}{k} + \frac{a\bar{x}_2}{(b+\bar{x}_1)^2} \right] \bar{x}_1$$

$$\det J_{\bar{E}} = \bar{x}_1 \bar{x}_2 \left(+ \frac{a}{b+\bar{x}_1} \right) \left(\frac{eab}{(b+\bar{x}_1)^2} \right) > 0$$

Il segno di $\text{tr}\, J_{\bar{E}}$ determina la stabilità di $J_{\bar{E}}$

All'equilibrio $\bar{E} \in$ parabola $\rightarrow \bar{x}_2 = \frac{r}{a}\left(1 - \bar{x}_1/k\right)(b + \bar{x}_1)$

$$\Rightarrow \text{tr}\, J_{\bar{E}} = \left[-\frac{r}{k} + \frac{r\left(1 - \frac{\bar{x}_1}{k}\right)\cancel{(b+\bar{x}_1)}}{(b+\bar{x}_1)^{\cancel{2}}} \right] \bar{x}_1$$

$$\text{segno}\left(\text{tr}\, J_{\bar{E}}\right) \propto -1 + \frac{k - \bar{x}_1}{b + \bar{x}_1}$$

$\mathrm{tr}\, J_{\bar{E}} < 0 \qquad -1 + \dfrac{k - \bar{x}_1}{b + \bar{x}_1} < 0$

$-b - \bar{x}_1 + k - \bar{x}_1 < 0$

$\bar{x}_1 > \dfrac{k-b}{2} \implies \begin{cases} \underbrace{\dfrac{mb}{ea-m}}_{\bar{x}_1} > \dfrac{k-b}{2} \\ \underbrace{\dfrac{mb}{ea-m} < k}_{\bar{x}_2 > 0} \end{cases}$

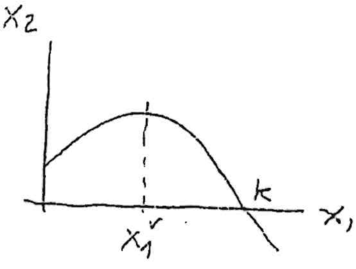
$\implies \dfrac{k-b}{2} < \dfrac{mb}{ea-m} < k \qquad \bar{E}$ è as. stab.

$\mathrm{tr}\, J_{\bar{E}} > 0 \quad \dots \qquad \dfrac{mb}{ea-m} < \dfrac{k-b}{2} \qquad \bar{E}$ è instabile

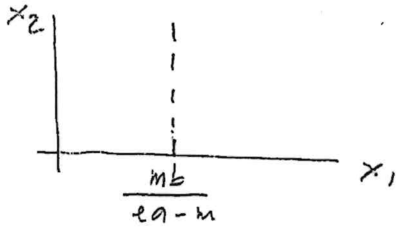
Riassumendo :

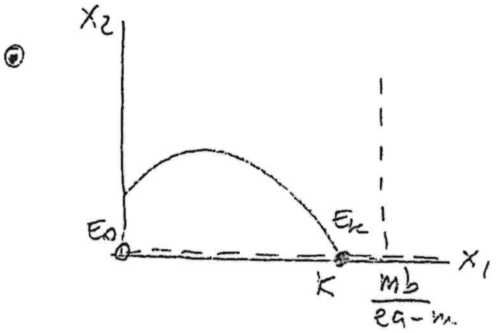
$$\dot{x}_1 = 0 \underset{\text{non banale}}{\rightsquigarrow} x_2 = \frac{r}{a}(b+x_1)\left(1-\frac{x_1}{k}\right)$$

$$x_1^v = \frac{k-b}{2}$$



$$\dot{x}_2 = 0 \underset{\text{non banale}}{\longrightarrow} x_1 = \frac{mb}{ea-m}$$

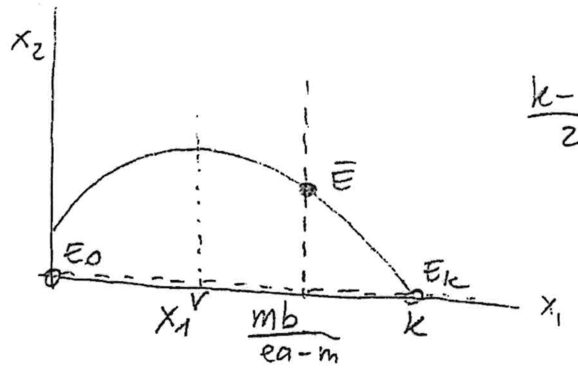




$$\frac{mb}{ea-m} > k$$

$E_k = (k;0)$ stab

$E_0 = (0\ 0)$ sella

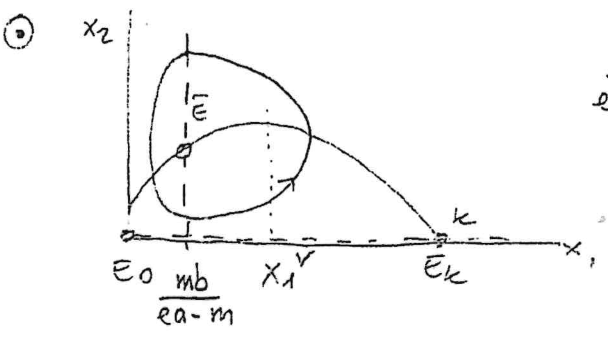


$$\frac{k-b}{2} < \frac{mb}{ea-m} < k$$

$E_k = (k,0)$ sella

$E_0 = (0,0)$ sella

$\bar{E} = (\bar{x}_1, \bar{x}_2)$ stabile



$$\frac{mb}{ea-m} < \frac{k-b}{2}$$

$E_k = (k,0)$ sella

$E_0 = (0,0)$ sella

$\bar{E} = (\bar{x}_1, \bar{x}_2)$ instabile

$\exists$ ciclo stabile

Modello di competizione interspecifica

x_1 = densità di batteri utili

x_2 = densità di batteri nocivi

$$\dot{x}_1 = r_1 x_1 \left(1 - \frac{x_1}{k_1}\right) - \alpha_{12} x_1 x_2 \longrightarrow$$

$$\dot{x}_2 = r_2 x_2 \left(1 - \frac{x_2}{k_2}\right) - \alpha_{21} x_1 x_2 \longrightarrow$$

extra-mortalità per competizione interspecifica

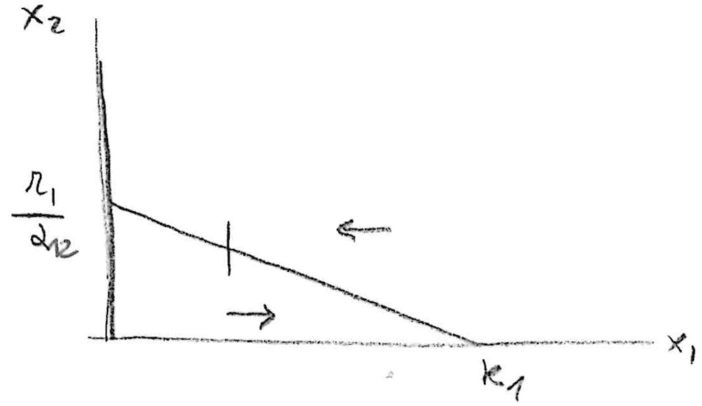
Determinare isocline, equilibri, traiettorie, stabilità

$$\dot{x}_1 = x_1 \left[r_1 \left(1 - \frac{x_1}{k_1}\right) - \alpha_{12} x_2 \right] = x_1 F_1(x_1, x_2)$$

$$\dot{x}_2 = x_2 \left[r_2 \left(1 - \frac{x_2}{k_2}\right) - \alpha_{21} x_1 \right] = x_2 F_2(x_1, x_2)$$

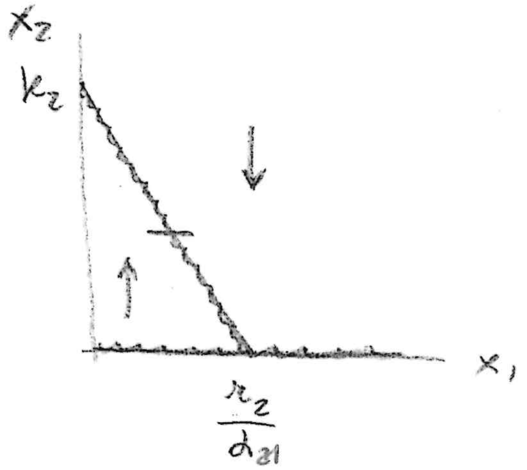
<u>Isocline</u> $\dot{x}_1 = 0$

- $x_1 = 0$
- $x_2 = \frac{r_1}{\alpha_{12}} \left(1 - \frac{x_1}{k_1}\right) \rightarrow F_1 = 0$



Isocline $\dot{x}_2 = 0$

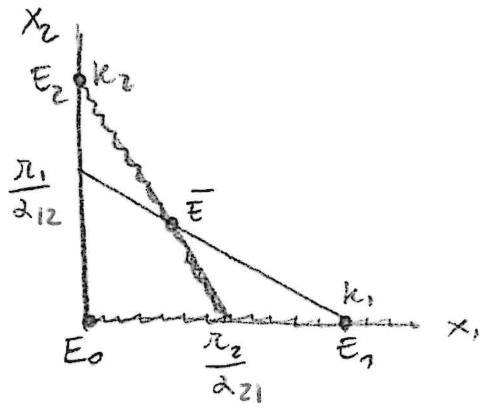
$$\dot{x}_2 = 0 \quad \begin{cases} x_2 = 0 \\ x_1 = \dfrac{r_2}{\alpha_{21}}\left(1 - \dfrac{x_2}{k_2}\right) \rightsquigarrow F_2 = 0 \end{cases}$$



Equilibri $\cap$ isocline $\dot{x}_1 = 0$ e $\dot{x}_2 = 0$

- $\begin{cases} k_1 > \dfrac{r_2}{\alpha_{21}} \\ k_2 > \dfrac{r_1}{\alpha_{12}} \end{cases}$

(α_{ij} grande)



$E_0 = (0, 0)$

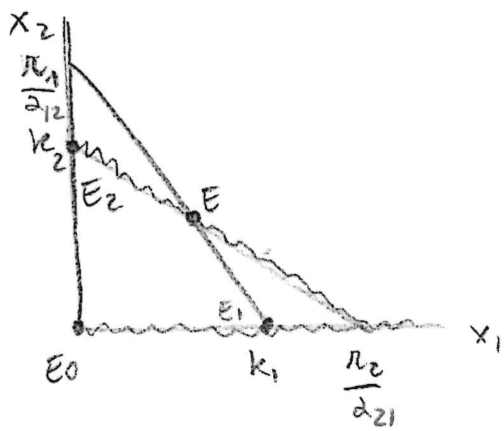
$E_1 = (k_1, 0)$

$E_2 = (0, k_2)$

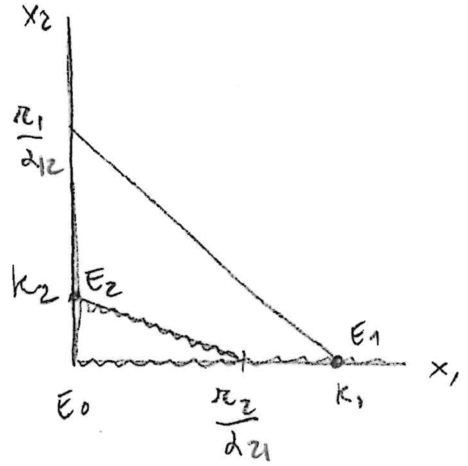
$\bar{E} = (\bar{x}_1, \bar{x}_2)$ t.c. $\begin{cases} F_1 = 0 \\ F_2 = 0 \end{cases}$

- $\begin{cases} k_1 < \dfrac{r_2}{\alpha_{21}} \\ k_2 < \dfrac{r_1}{\alpha_{12}} \end{cases}$

(α_{ij} piccolo)



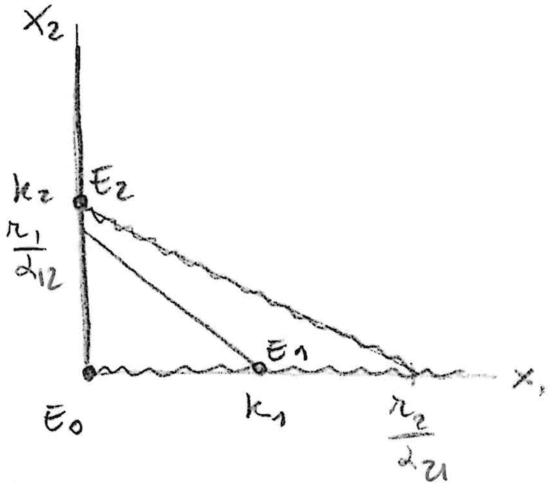
$$\bullet \begin{cases} k_1 > \frac{r_2}{\alpha_{21}} \\ k_2 < \frac{r_1}{\alpha_{12}} \end{cases}$$



$\nexists \bar{E}$

$E_0 = (0,0)$
$E_1 = (k_1, 0)$
$k_2 = (0, k_2)$

$$\bullet \begin{cases} k_1 < \frac{r_2}{\alpha_{21}} \\ k_2 > \frac{r_1}{\alpha_{12}} \end{cases}$$



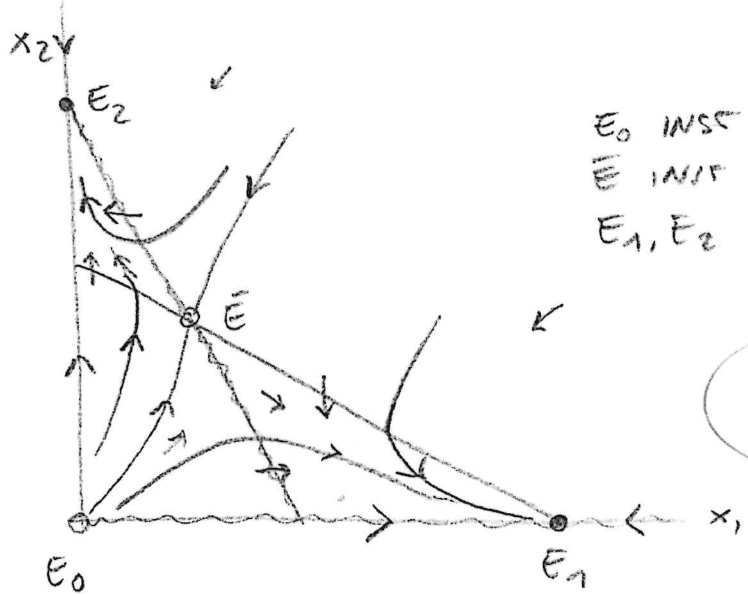
$\nexists \bar{E}$

$E_0 = (0,0)$
$E_1 = (k_1, 0)$
$E_2 = (0, k_2)$

<u>Traiettorie</u>

<u>NOTA</u>: Gli assi sono invarianti

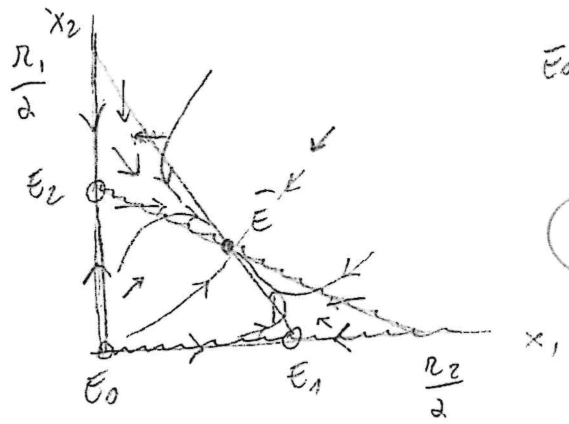
$$\bullet \begin{cases} k_1 > \frac{r_2}{\alpha_{21}} \\ k_2 > \frac{r_1}{\alpha_{12}} \end{cases}$$



E_0 INST (REP)
$\bar{E}$ INST (SELLA)
E_1, E_2 STAB

Esclusione competitiva

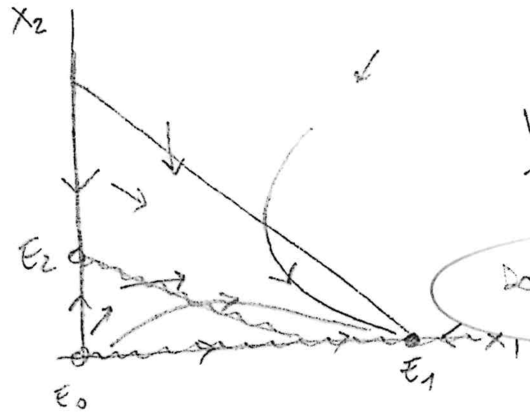
$$\begin{cases} k_1 < \dfrac{r_2}{\alpha_{21}} \\ k_2 < \dfrac{r_1}{\alpha_{12}} \end{cases}$$



E_0, E_1, E_2 INST
$\bar{E}$ STAB

Coesistenza competitiva

$$\begin{cases} k_1 > \dfrac{r_2}{\alpha_{21}} \\ k_2 < \dfrac{r_1}{\alpha_{12}} \end{cases}$$

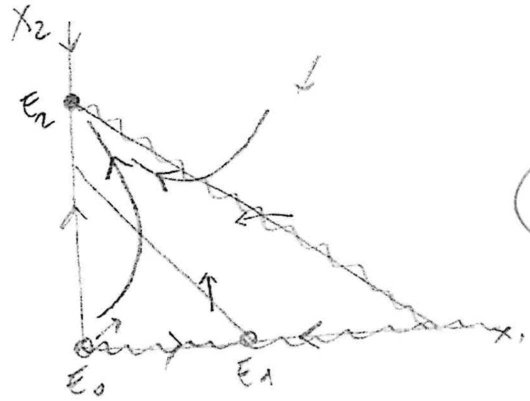


E_0, E_2 INST
E_1 STAB

Vince la competizione la specie con isoclina "più alta"

DOMINANZA 1

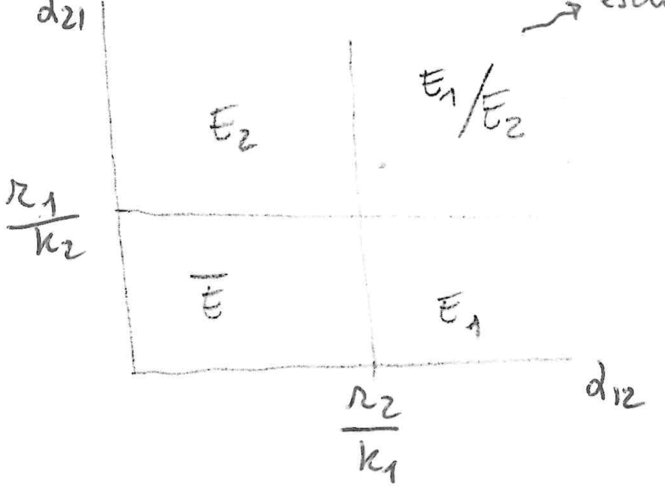
$$\begin{cases} k_1 < \dfrac{r_2}{\alpha_{21}} \\ k_2 > \dfrac{r_1}{\alpha_{12}} \end{cases}$$



E_0, E_1 INST
E_2 STAB

DOMINANZA 2

EQ. STABILI



→ esclusione competitiva

Studio della stabilità via linearizzazione

$$\dot{x}_1 = x_1 F_1(x_1, x_2) = f_1(x_1, x_2)$$
$$\dot{x}_2 = x_2 F_2(x_1, x_2) = f_2(x_1, x_2)$$

$$J = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{vmatrix} = \begin{vmatrix} F_1 + x_1 \frac{\partial F_1}{\partial x_1} & x_1 \frac{\partial F_1}{\partial x_2} \\ x_2 \frac{\partial F_2}{\partial x_1} & F_2 + x_2 \frac{\partial F_2}{\partial x_2} \end{vmatrix}$$

$$\frac{\partial F_1}{\partial x_1} = -\frac{r_1}{k_1} \qquad \frac{\partial F_1}{\partial x_2} = -\alpha_{12}$$

$$\frac{\partial F_2}{\partial x_1} = -\alpha_{21} \qquad \frac{\partial F_2}{\partial x_2} = -\frac{r_2}{k_2}$$

- $E_0 = (0,0) \rightarrow J\Big|_{E_0} = \begin{vmatrix} F_1(0,0) & 0 \\ 0 & F_2(0,0) \end{vmatrix} = \begin{vmatrix} r_1 & 0 \\ 0 & r_2 \end{vmatrix}$

  $\lambda_1 = r_1 \quad \lambda_2 = r_2 \quad \lambda_i > 0 \rightarrow$ INST (REPULS)

- $E_1(k_1, 0) \rightarrow J\Big|_{E_1} = \begin{vmatrix} -r_1 & * \\ 0 & r_2 - \alpha_{21} k_1 \end{vmatrix}$ $\quad \lambda_1 = -r_1 < 0$, $\quad \lambda_2 = r_2 - \alpha_{21} k_1$

  $F_1(E_1) = 0$

  $r_2 - \alpha_{21} k_1 < 0 \rightarrow k_1 > \frac{r_2}{\alpha_{21}} \rightarrow \lambda_2 < 0 \Rightarrow E_1$ STAB

  $r_2 - \alpha_{21} k_1 > 0 \rightarrow k_1 < \frac{r_2}{\alpha_{21}} \rightarrow \lambda_2 > 0 \Rightarrow E_1$ INST

- $E_2 = (0, k_2) \rightsquigarrow$ come sopra $\rightsquigarrow$ $k_2 > \frac{r_1}{\alpha_{12}} \Rightarrow E_2$ STAB

  $k_2 < \frac{r_1}{\alpha_{12}} \Rightarrow E_2$ INST

- $\bar{E} = (\bar{x}_1, \bar{x}_2) \rightarrow J\Big|_{\bar{E}} = \begin{vmatrix} \bar{x}_1\left(-\frac{r_1}{k_1}\right) & \bar{x}_1(-\alpha_{12}) \\ \bar{x}_2(-\alpha_{21}) & \bar{x}_2\left(-\frac{r_2}{k_2}\right) \end{vmatrix}$ con $\bar{x}_1, \bar{x}_2 > 0$

  $F_1 = F_2 = 0$

$tr < 0$

$$\det = \bar{x}_1 \bar{x}_2 \left(\frac{r_1 r_2}{k_1 k_2} - \alpha_{12} \alpha_{21} \right)$$

$$\det > 0 \Rightarrow \frac{r_1 r_2}{k_1 k_2} - \alpha_{12}\alpha_{21} > 0 \Rightarrow \bar{E} \text{ è STAB}$$

$\exists \bar{E}$ solo per $\begin{cases} k_1 < \frac{r_2}{\alpha_{21}} \\ k_2 < \frac{r_1}{\alpha_{12}} \end{cases} \rightsquigarrow \frac{r_1 r_2}{\alpha_{12}\alpha_{21}} > k_1 k_2 \rightsquigarrow \frac{r_1 r_2}{k_1 k_2} > \alpha_{12}\alpha_{21} \rightsquigarrow \det > 0$

$$\Downarrow$$

$\bar{E}$ STAB

oppure per $\begin{cases} k_1 > \frac{r_2}{\alpha_{21}} \\ k_2 > \frac{r_1}{\alpha_{12}} \end{cases} \rightsquigarrow \dots \rightsquigarrow \det < 0 \Rightarrow \bar{E}$ INST

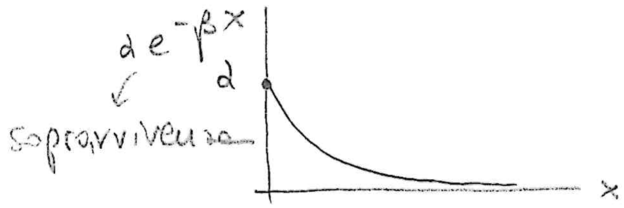
Modello di Ricker $\rightarrow$ crescita di una singola risorsa a tempo discreto $x(t)$

$\downarrow$

Salmoni dell'Oceano Pacifico (popolazione semelpara)

NOTA: $\alpha > 1$
x piccolo $e^{-\beta x} \sim 1$
$x(t+1) = \alpha x(t)$ e $x(t)$ cresce
se $\alpha < 1$ $x(t)$ decresce

$$x(t+1) = \alpha x(t) e^{-\beta x(t)} = f(x(t))$$



Equilibri $\quad x(t+1) = x(t) = \bar{x}$ $\qquad x = \alpha x e^{-\beta x}$

$\bar{x} = 0$

$1 = \alpha e^{-\beta x}$

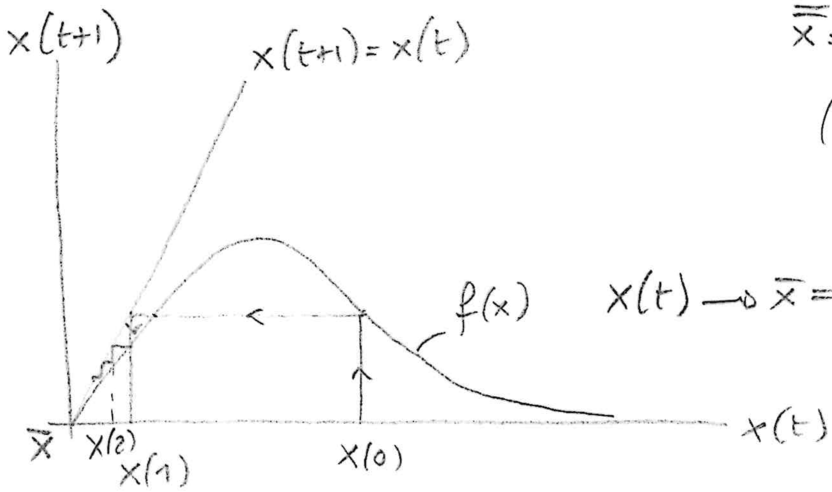
$\downarrow$

$\bar{\bar{x}} = \frac{1}{\beta} \ln \alpha$ (con $\alpha > 1$)

Diagramma di Moran

$\alpha < 1$ ($\nexists \bar{\bar{x}}$)

($f'(0) = \alpha < 1$)



$x(t) \rightarrow \bar{x} = 0$

$\alpha > 1$? Dipende! $\quad 1 < \alpha < e^2 \Rightarrow x(t) \rightarrow \bar{\bar{x}}$

$\alpha > e^2 \Rightarrow x(t) \rightarrow$ qualcosa di più complicato

- ciclo
- caos

vedi simulatore

Con la linearizzazione:

$$J = \frac{\partial f}{\partial x} = \alpha e^{-\beta x} - \alpha x e^{-\beta x} \beta = \alpha e^{-\beta x}(1 - \beta x)$$

$J_{\bar{x}=0} = \alpha = \lambda \qquad \alpha<1 \quad \bar{x}=0 \quad A.S.$

$\qquad\qquad\qquad\qquad \alpha>1 \quad \bar{x}=0 \quad INST$

$J_{\bar{\bar{x}}=\frac{1}{\beta}\ln\alpha} = \cancel{\alpha}\, 1\cdot\left(1-\beta\frac{1}{\beta}\ln\alpha\right) = 1-\ln\alpha = \lambda$

$(\alpha>1)$

$\downarrow$ all'equilibrio

$\alpha e^{-\beta x} = 1$

$\bar{\bar{x}}$ è A.S. se $\quad -1<1-\ln\alpha<1 \rightsquigarrow 0<\ln\alpha<2 \rightsquigarrow 1<\alpha<e^2$

$\bar{\bar{x}}$ è INST se $\alpha>e^2$

Modelli di epidemie

SI — infetti, suscettibili

$S \xrightarrow{\alpha} I \xrightarrow{\beta}$

Immunizzazione permanente (morbillo/rosolia/varicella ...)

$$\dot{S} = -\alpha S I \quad \text{(contagio)}$$
$$\dot{I} = \alpha S I - \beta I \quad \text{(guarigione)}$$

$\alpha \downarrow$ con mascherine/vaccini

$\beta \uparrow$ con cure

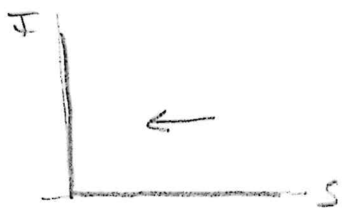
Equilibri

$$\dot{S} = 0 \; \begin{cases} S = 0 \\ I = 0 \end{cases}$$
$$\dot{I} = 0 \; \begin{cases} I = 0 \\ S = \frac{\beta}{\alpha} \end{cases}$$

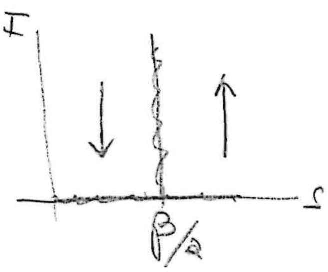
$\Rightarrow \exists \infty$ equilibri $(\bar{S}, 0)$

Isocline

$\dot{S} = 0$

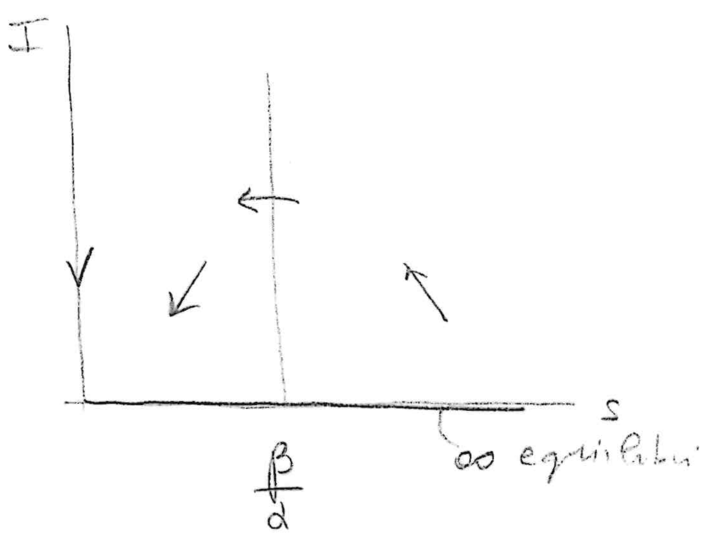


$\dot{I} = 0$



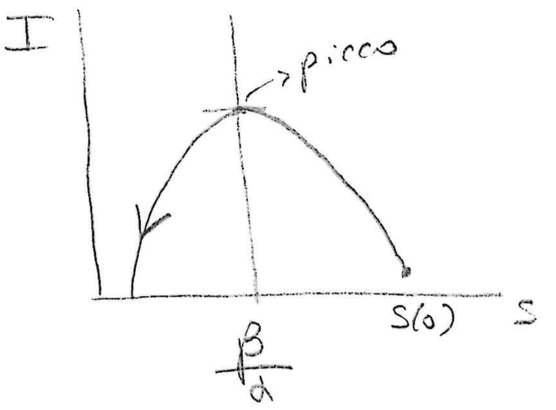
Equilibri $\Rightarrow \cap$ isocline $\Rightarrow I = 0 \;\; \forall S$

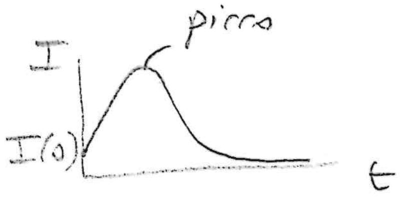
Vettore tangente



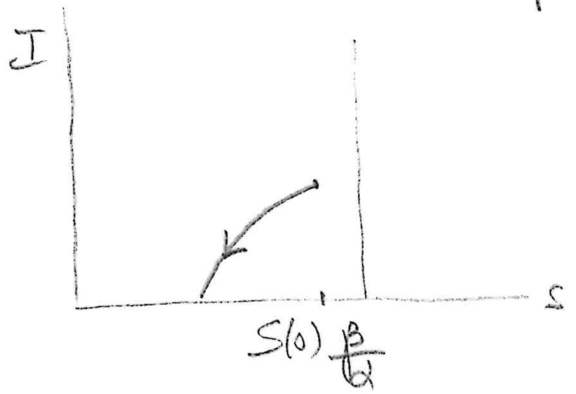
$S(0) > \frac{\beta}{\alpha} \rightarrow$ L'epidemia "parte"

$I(0) > 0$





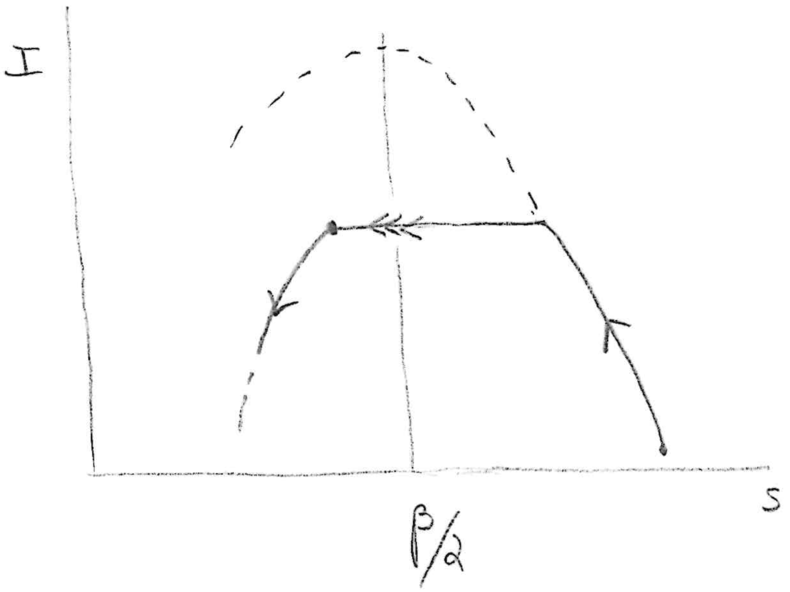
$S(0) < \frac{\beta}{\alpha} \rightarrow$ L'epidemia "non parte"



Per fare in modo che $S(0)$ sia $< \frac{\beta}{\alpha} \rightarrow$ β alto (cure)
α piccolo (mascherine, vaccini)

Lockdown?

$S\downarrow$ sotto β/α



Il sistema ha infiniti equilibri $(\bar{S}, 0)$. Di questi, sono instabili quelli con $\bar{S} > \beta/\alpha$, stabili (non asintoticamente) quelli con $\bar{S} < \beta/\alpha$