Hopf bif. in a control system.

$$u \longrightarrow \boxed{\frac{1}{(1+sT)^3}} \longrightarrow y$$

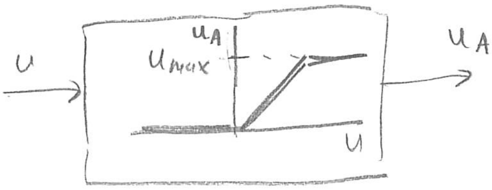
e.g. 3 identical tanks

goal: $y = y^0$ constant

control law: $u = y^0 + K(y^0 - y)$

↳ open loop
↳ closed loop
} control

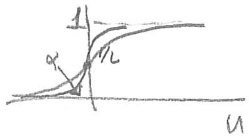
saturation



(A: actuated)

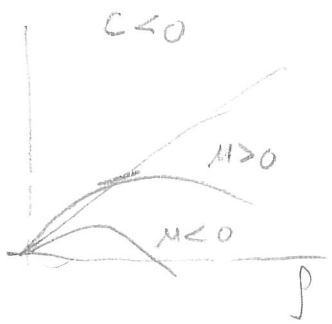
smooth saturation: $u_A = \text{step}(u)\cdot u - \text{step}(u-u_{max})(u-u_{max})$

$$\text{step}(u) = \frac{\exp(\alpha u)}{\exp(-\alpha u)+\exp(\alpha u)}$$



control system as stab $\Leftrightarrow K < 8$

Neimark Sacker normal form



$$\rho(t+1) = (1+\mu)\rho(t) + c\rho(t)^3$$

$$\vartheta(t+1) = \vartheta(t) + \alpha$$

$$x_1(t+1) = \rho(t+1)\cos\vartheta(t+1) = \left((1+\mu)\rho(t) + c\rho(t)^3\right)\left(\cos\vartheta(t)\cos\alpha - \sin\vartheta(t)\sin\alpha\right)$$

$$= (1+\mu)\cos\alpha\, x_1(t) - (1+\mu)\sin\alpha\, x_2(t)$$

$$+ c\left(\cos\alpha\, x_1(t) - \sin\alpha\, x_2(t)\right)\left(x_1(t)^2 + x_2(t)^2\right)$$

$$x_2(t+1) = \rho(t+1)\sin\vartheta(t+1) = \left((1+\mu)\rho(t) + c\rho(t)^3\right)\left(\sin\vartheta(t)\cos\alpha + \cos\vartheta(t)\sin\alpha\right)$$

$$= (1+\mu)\sin\alpha\, x_1(t) + (1+\mu)\cos\alpha\, x_2(t)$$

$$+ c\left(\sin\alpha\, x_1(t) + \cos\alpha\, x_2(t)\right)\left(x_1(t)^2 + x_2(t)^2\right)$$