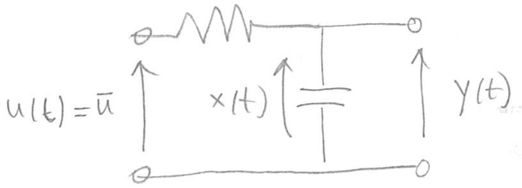
time dependency

$$R(t) = R_0(1+\varepsilon \sin \omega t) \qquad \omega = \frac{2\pi}{T}, \; T = 1 \text{ day}$$



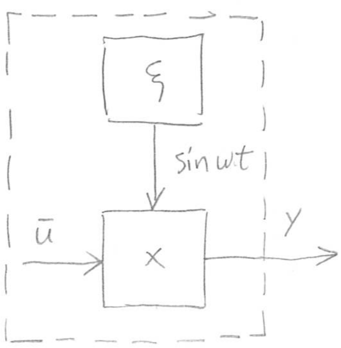
$$\dot{x} = \frac{1}{C}\,\frac{u-x}{R}, \quad y = x \quad \text{(low-pass filter)}$$

$$G(s) = \frac{1}{1+s\tau}, \quad \tau = RC : \text{time-constant}$$

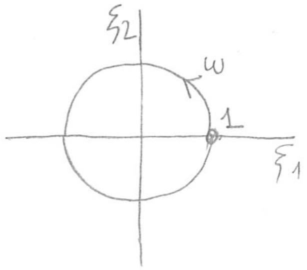
$$\dot{x} = \frac{\bar{u} - x}{R_0 C(1+\varepsilon \sin \omega t)} = f(x,t)$$

Note: the system is non-autonomous, because of the input u. However, constant inputs ($u(t)=\bar{u}$) are considered as model parameters and are often not mentioned explicitly. We can therefore consider the system as autonomous

time-independent extension

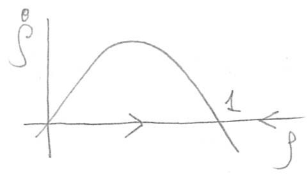


state $z = \begin{bmatrix} x \\ \xi \end{bmatrix}$



$$\xi_1(t) = \cos \omega t$$
$$\xi_2(t) = \sin \omega t$$

$$\dot{\rho} = \rho(1-\rho^2), \quad \xi_1 = \rho \cos \vartheta$$
$$\dot{\vartheta} = \omega, \quad \xi_2 = \rho \sin \vartheta$$



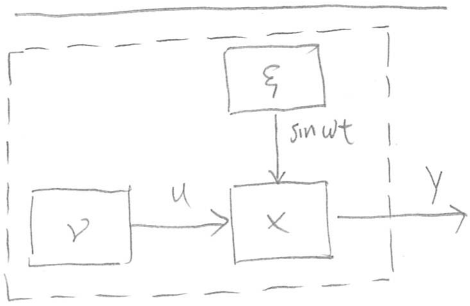
$$\rho(t) \xrightarrow[t\to\infty]{} 1$$
$$\vartheta(t) = \omega t$$

$$\dot{\xi}_1 = \dot{\rho}\cos\vartheta - \rho\omega \sin\vartheta$$
$$\dot{\xi}_2 = \dot{\rho}\sin\vartheta + \rho\omega\cos\vartheta$$

$$\dot{x} = \frac{\bar{u} - x}{R_0 C(1+\varepsilon \xi_2)} = f(x, \xi_2)$$

$$\dot{\xi}_1 = \xi_1 - \omega\xi_2 - (\xi_1^2 + \xi_2^2)\xi_1 = h_1(\xi_1, \xi_2)$$
$$\dot{\xi}_2 = \omega\xi_1 + \xi_2 - (\xi_1^2 + \xi_2^2)\xi_2 = h_2(\xi_1, \xi_2)$$

non autonomous case



state $z = \begin{bmatrix} x \\ \xi \\ \nu \end{bmatrix}$

$$\dot{x} = f(x, u, t)$$
$$\Downarrow$$
$$\dot{x} = f(x, \xi_2, \nu)$$
$$\dot{\xi} = h(\xi)$$
$$\dot{\nu} = q(\nu)$$