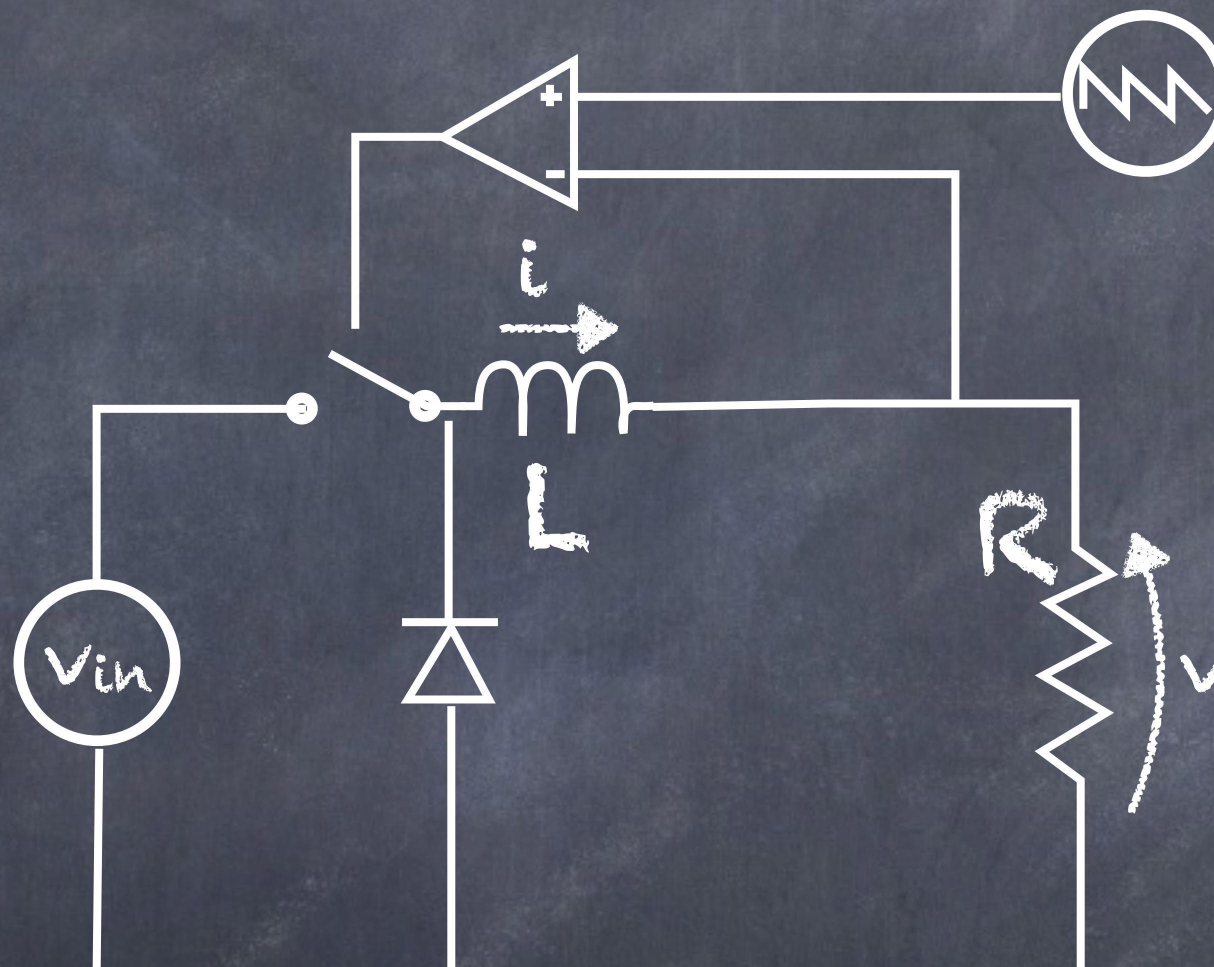
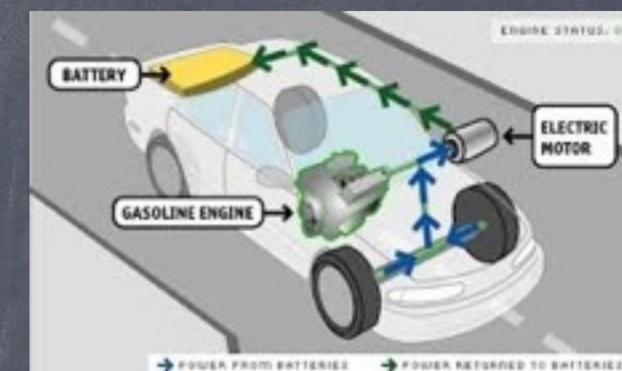
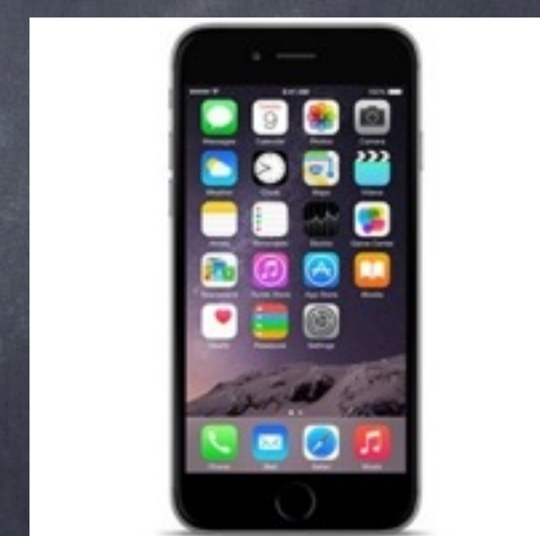
Buck converter

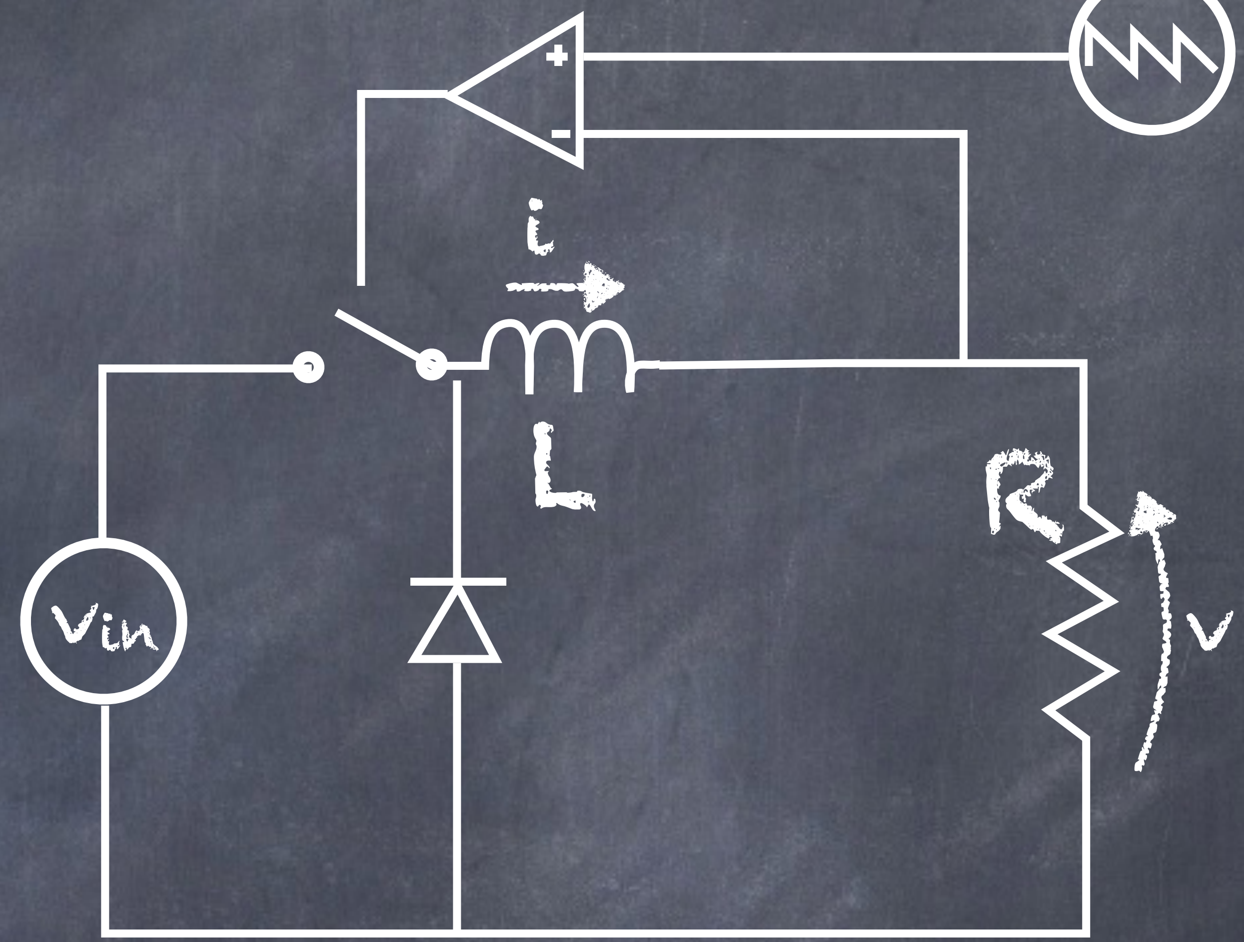






$$L\frac{di}{dt} = v$$

$$C\frac{dv}{dt} = i$$



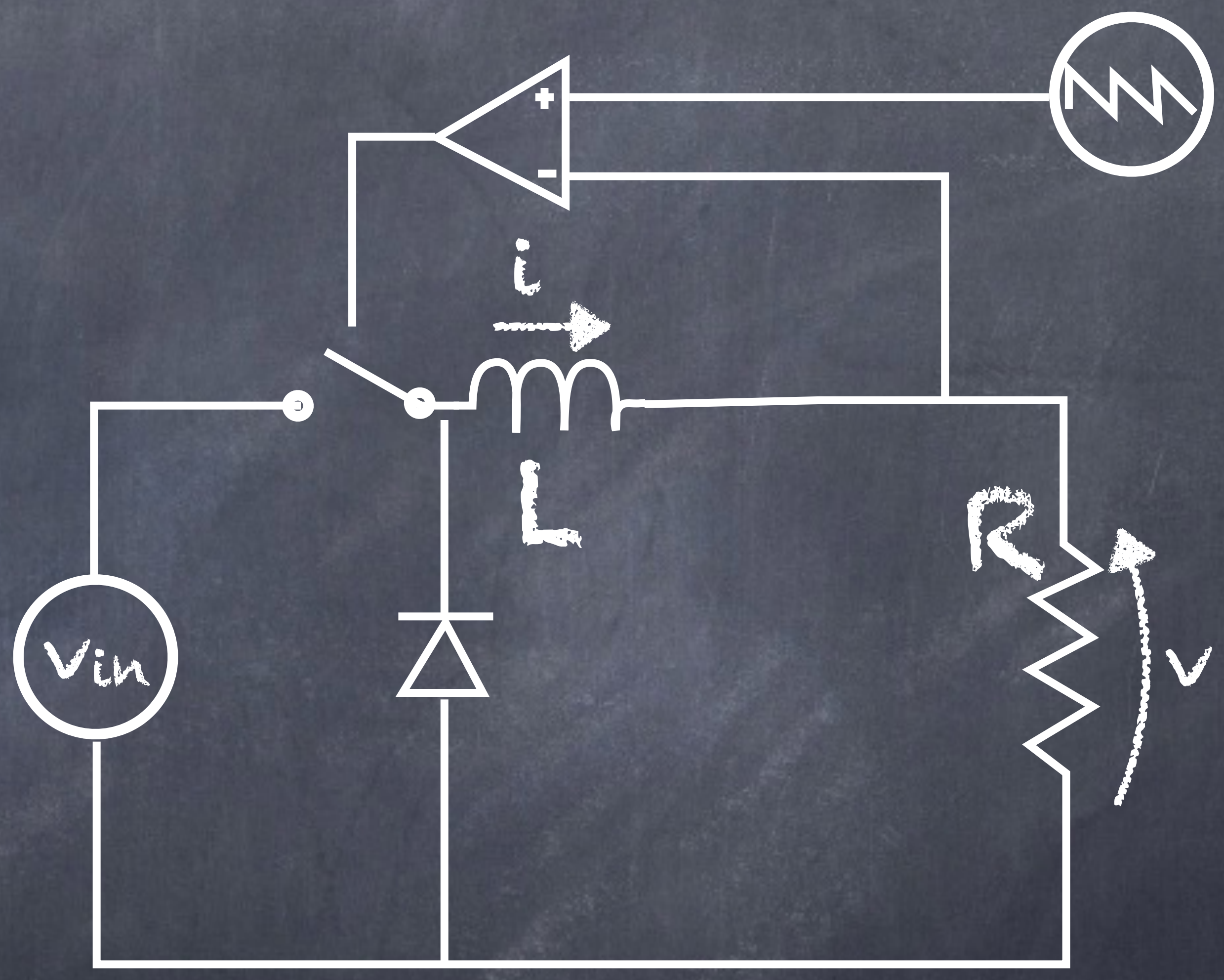
The switch is opened when v is greater than the sawtooth threshold, closed otherwise

When the switch is open (off)

$$\frac{di}{dt} = -\frac{v}{L}$$

therefore

$$\frac{dv}{dt} = -v\frac{R}{L}$$

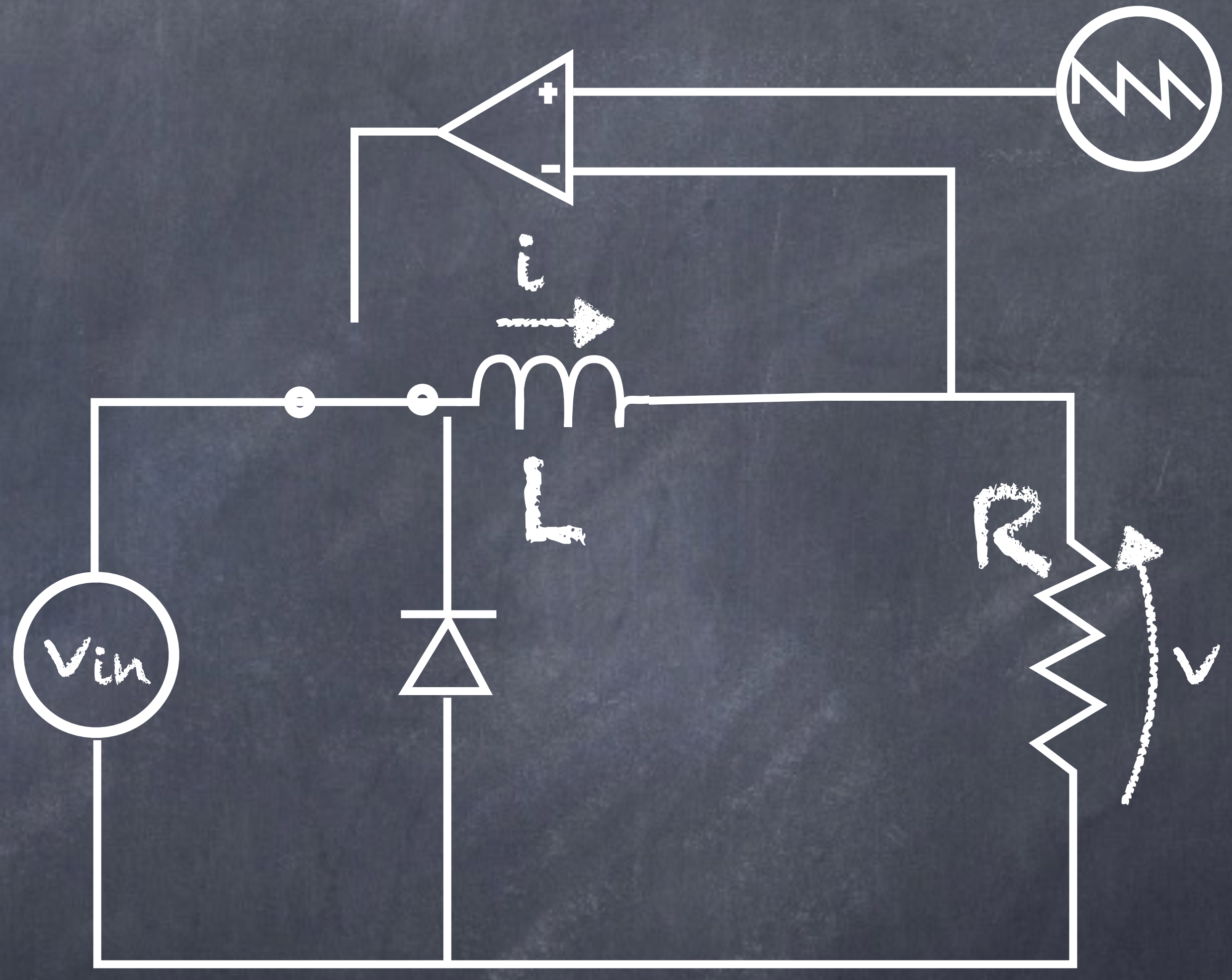


When the switch is closed (on)

$$\frac{di}{dt} = -\frac{v_{in} - v}{L}$$

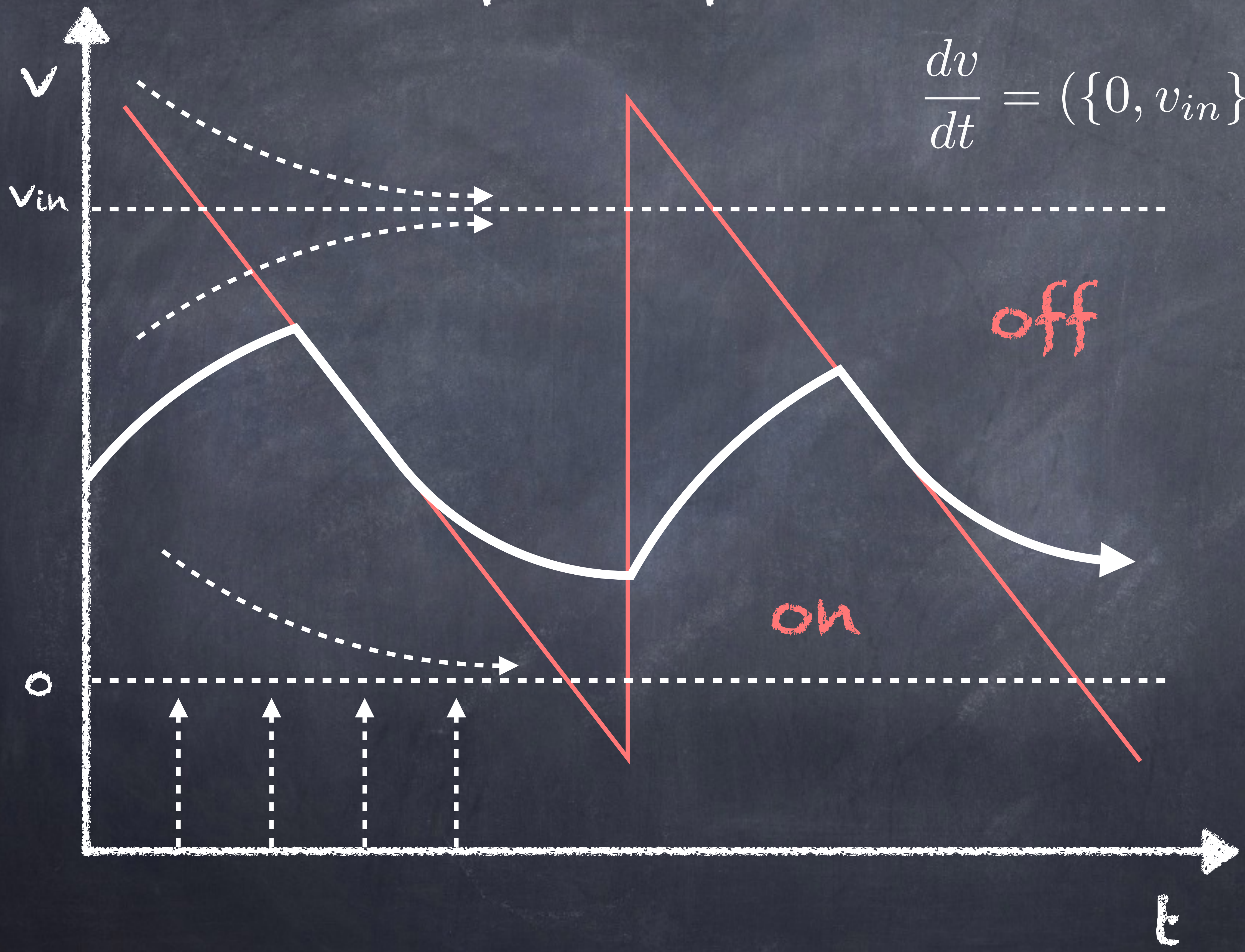
therefore

$$\frac{dv}{dt} = (v_{in} - v)\frac{R}{L}$$

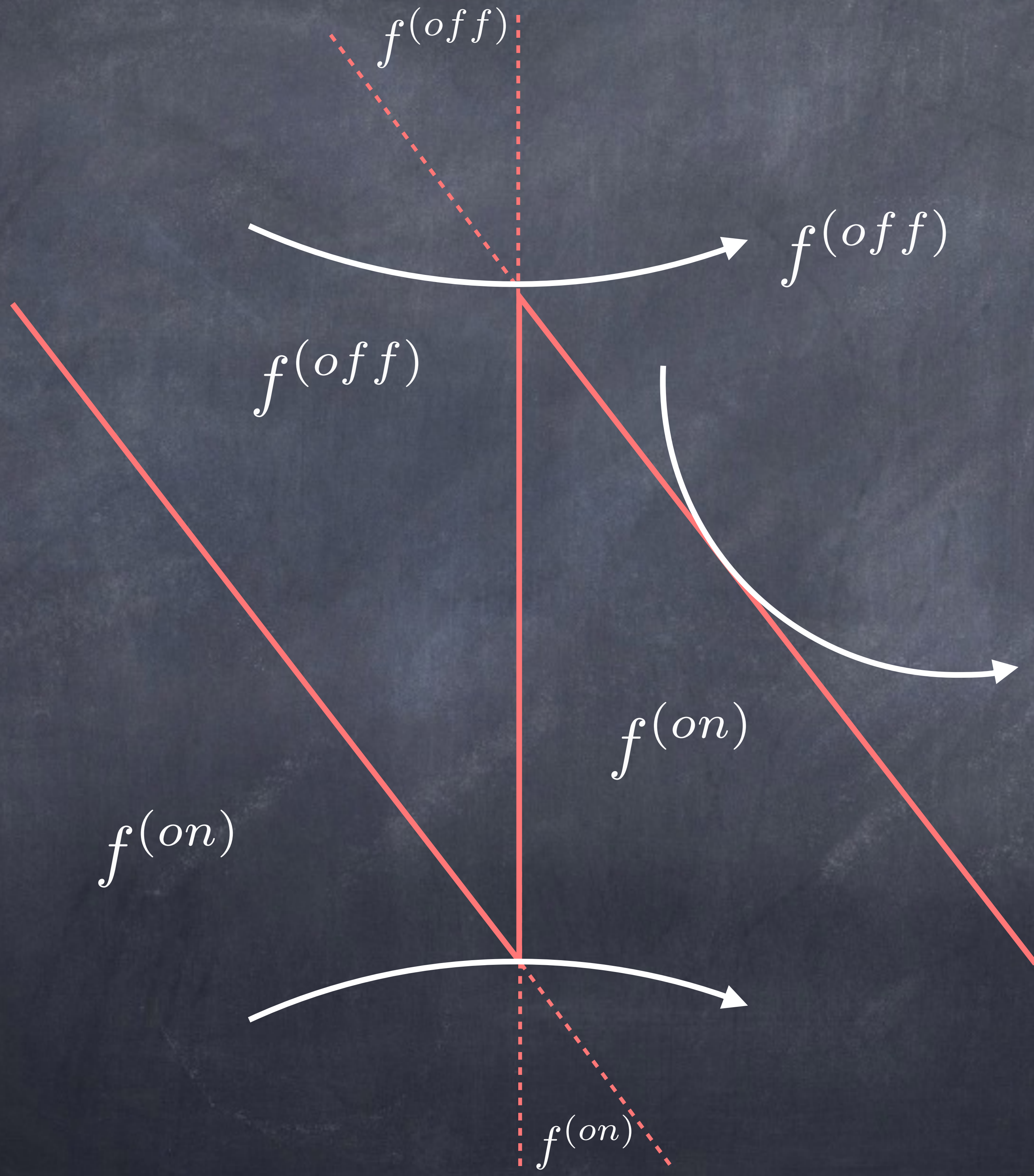


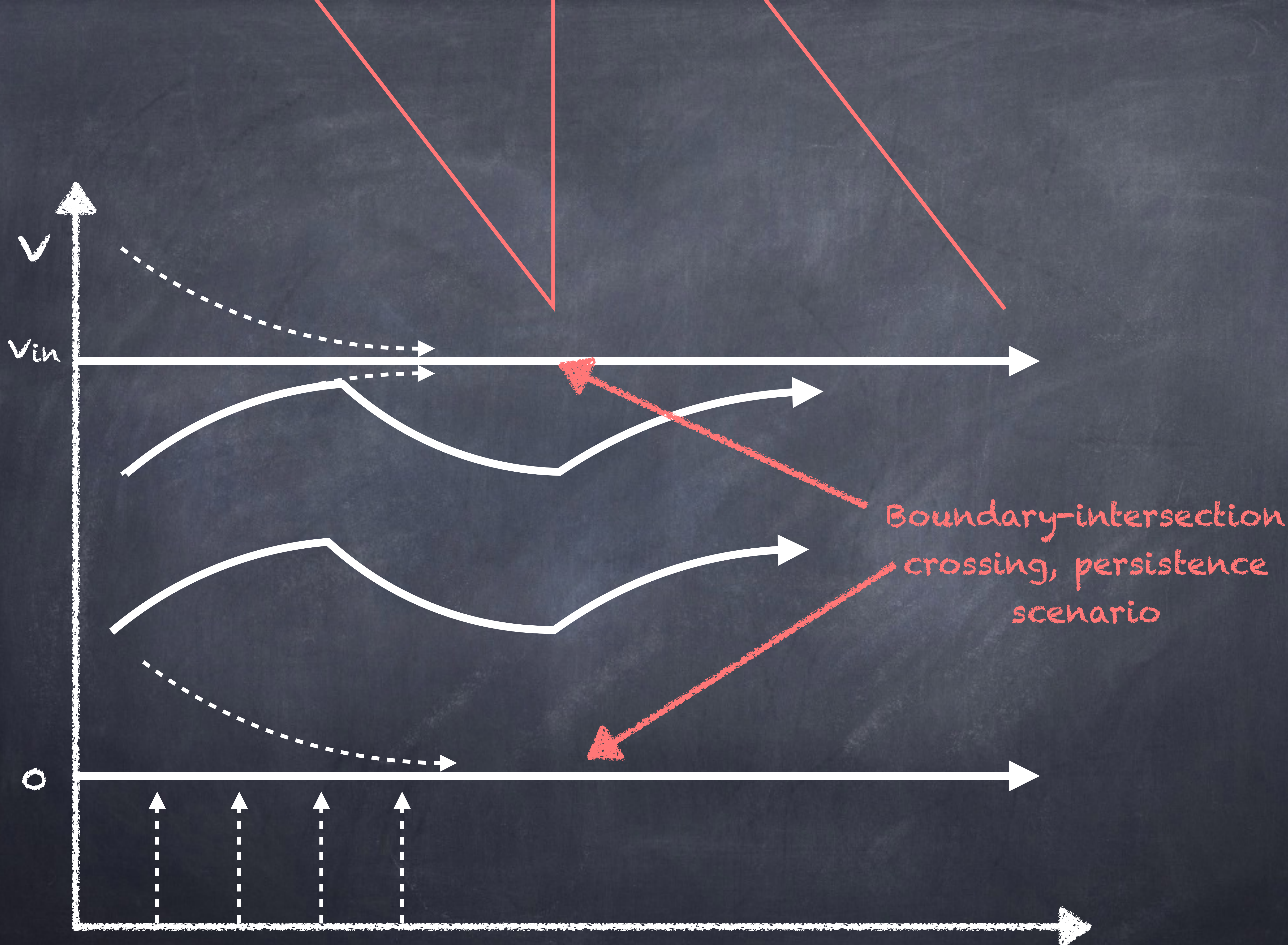
phase portrait

$$\frac{dv}{dt} = (\{0, v_{in}\} - v)\frac{R}{L}$$

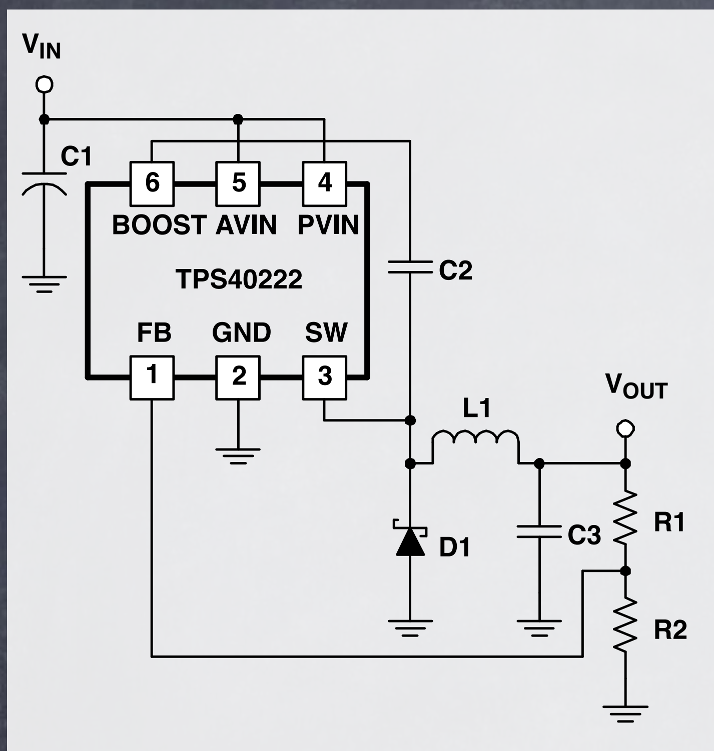


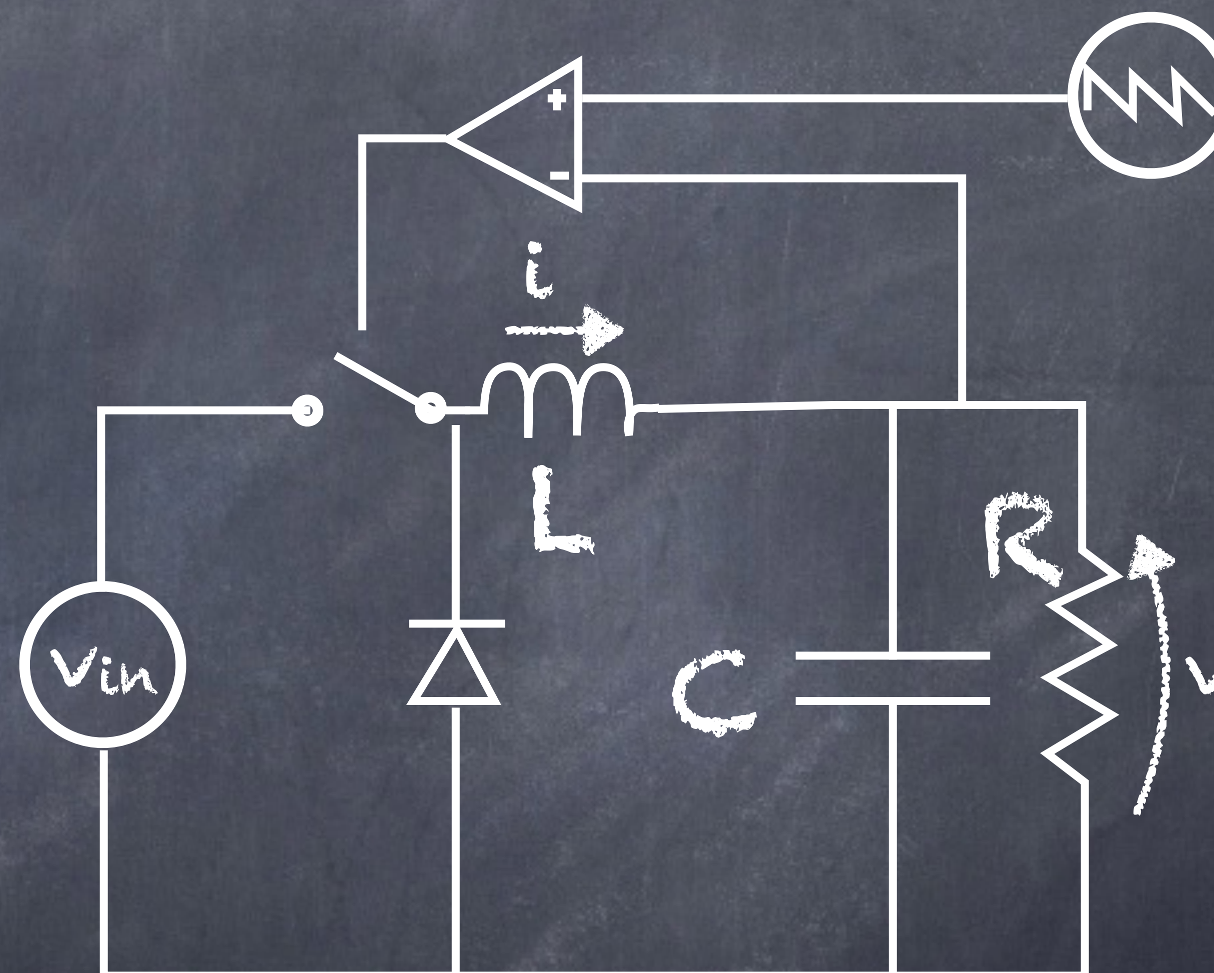
possible nonsmooth bifurcations




V
Vin
0
Boundary-intersection crossing, persistence scenario

a better model (with an output capacitor to reduce the ripple)





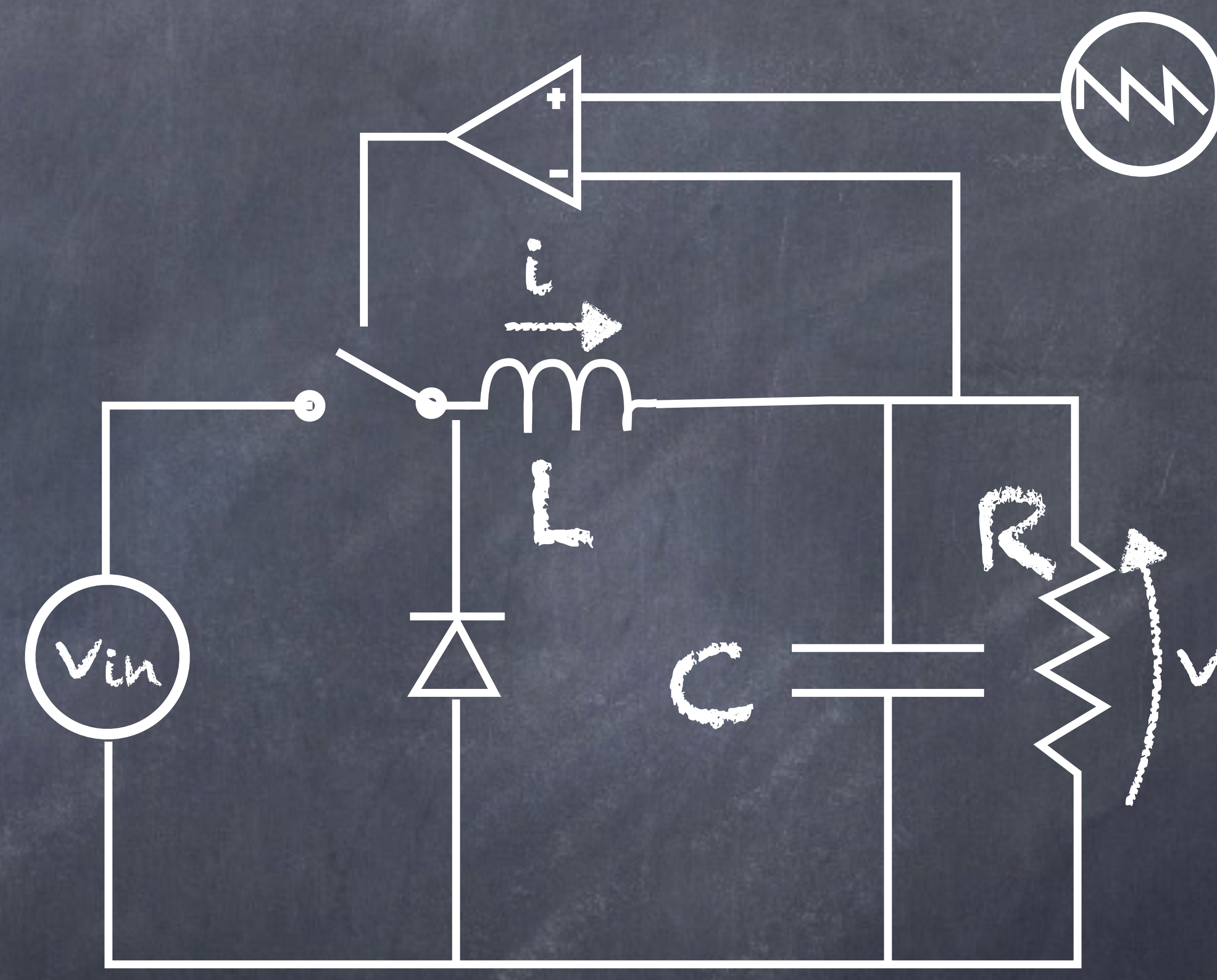
$$\frac{di}{dt} = -\frac{v}{L}$$

$$\frac{dv}{dt} = \frac{i}{C} - \frac{v}{RC}$$

When the switch is open (off) and i is zero

$$\frac{di}{dt} = \max\left(-\frac{v}{L}, 0\right)$$

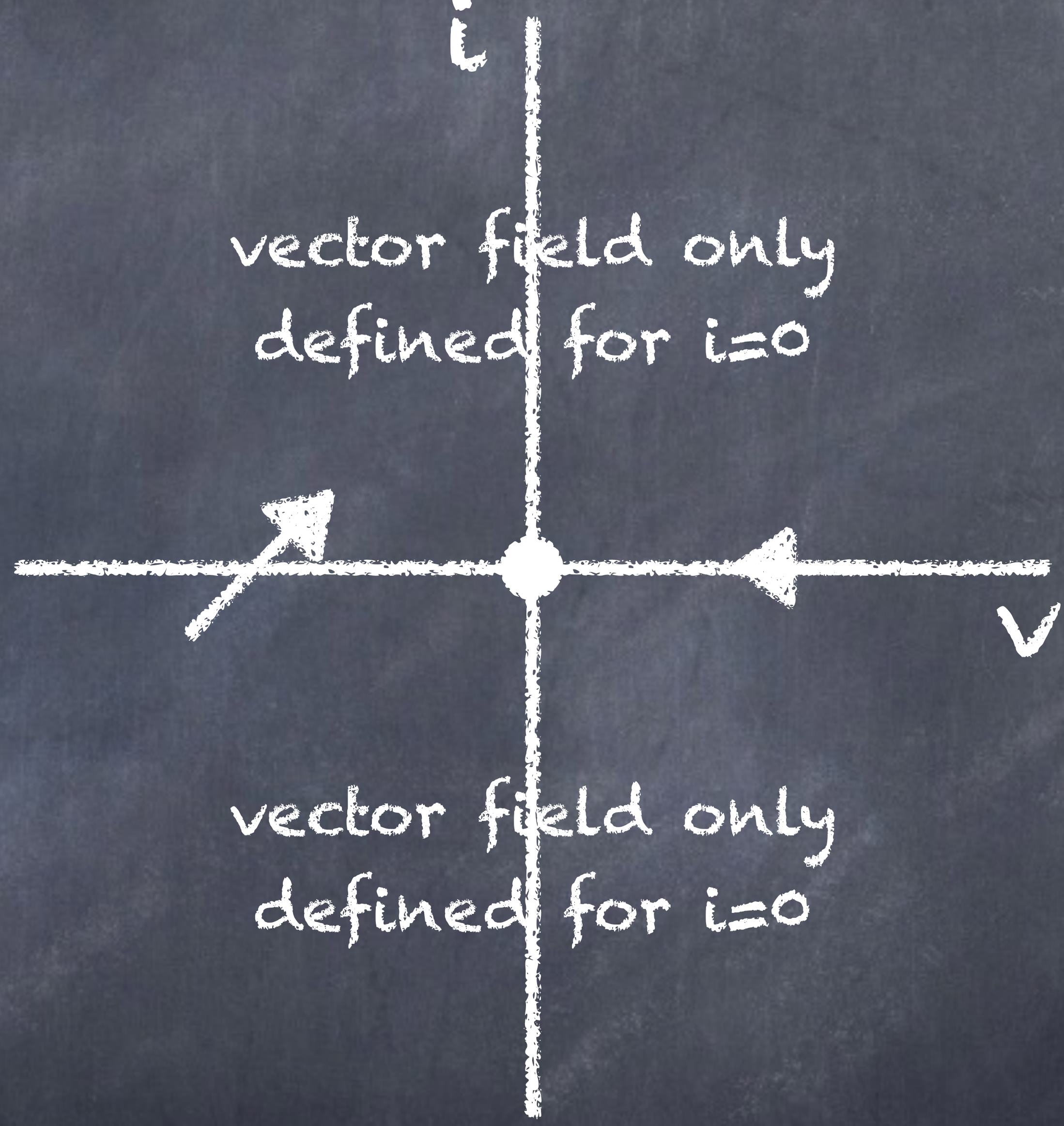
$$\frac{dv}{dt} = \frac{i}{C} - \frac{v}{RC}$$



When the switch is open (off) and i is zero

$$\frac{di}{dt} = \max\left(-\frac{v}{L}, 0\right)$$

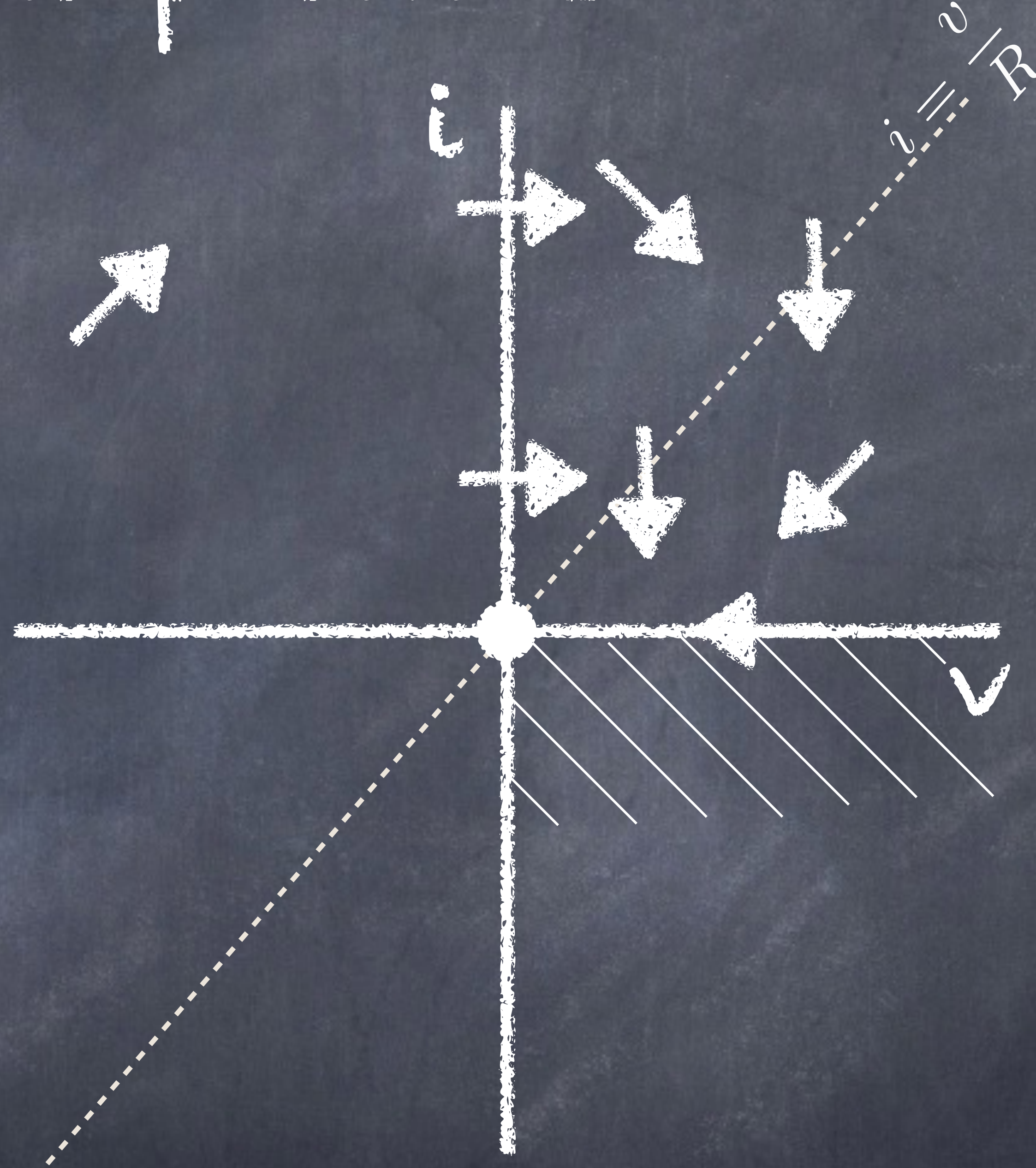
$$\frac{dv}{dt} = -\frac{v}{RC}$$



When the switch is open (off) and i is positive

$$\frac{di}{dt} = -\frac{v}{L}$$

$$\frac{dv}{dt} = \frac{i}{C} - \frac{v}{RC}$$

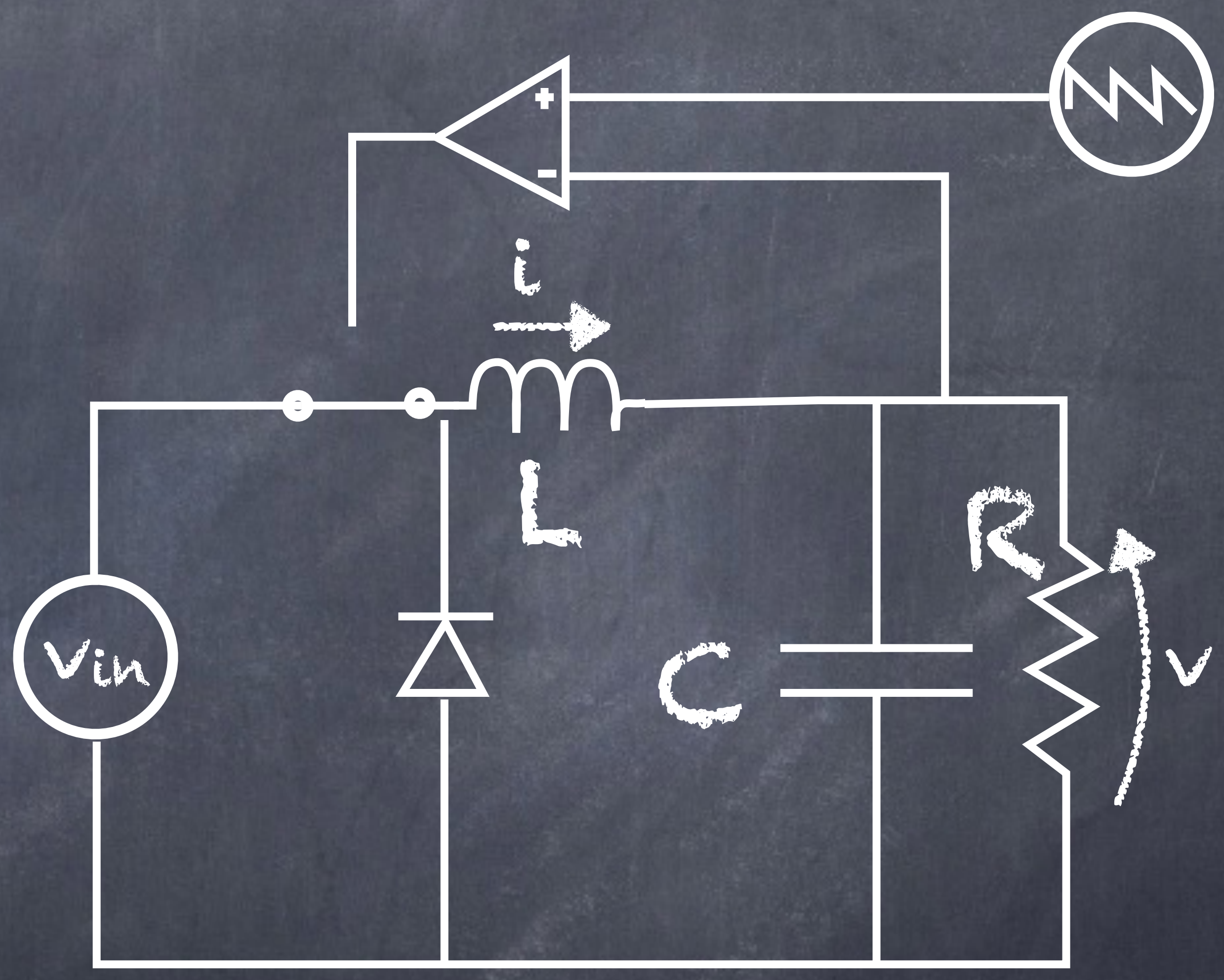


eigenvalues: $$-\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$$

When the switch is closed (on)

$$\frac{di}{dt} = \frac{v_{in}}{L} - \frac{v}{L}$$

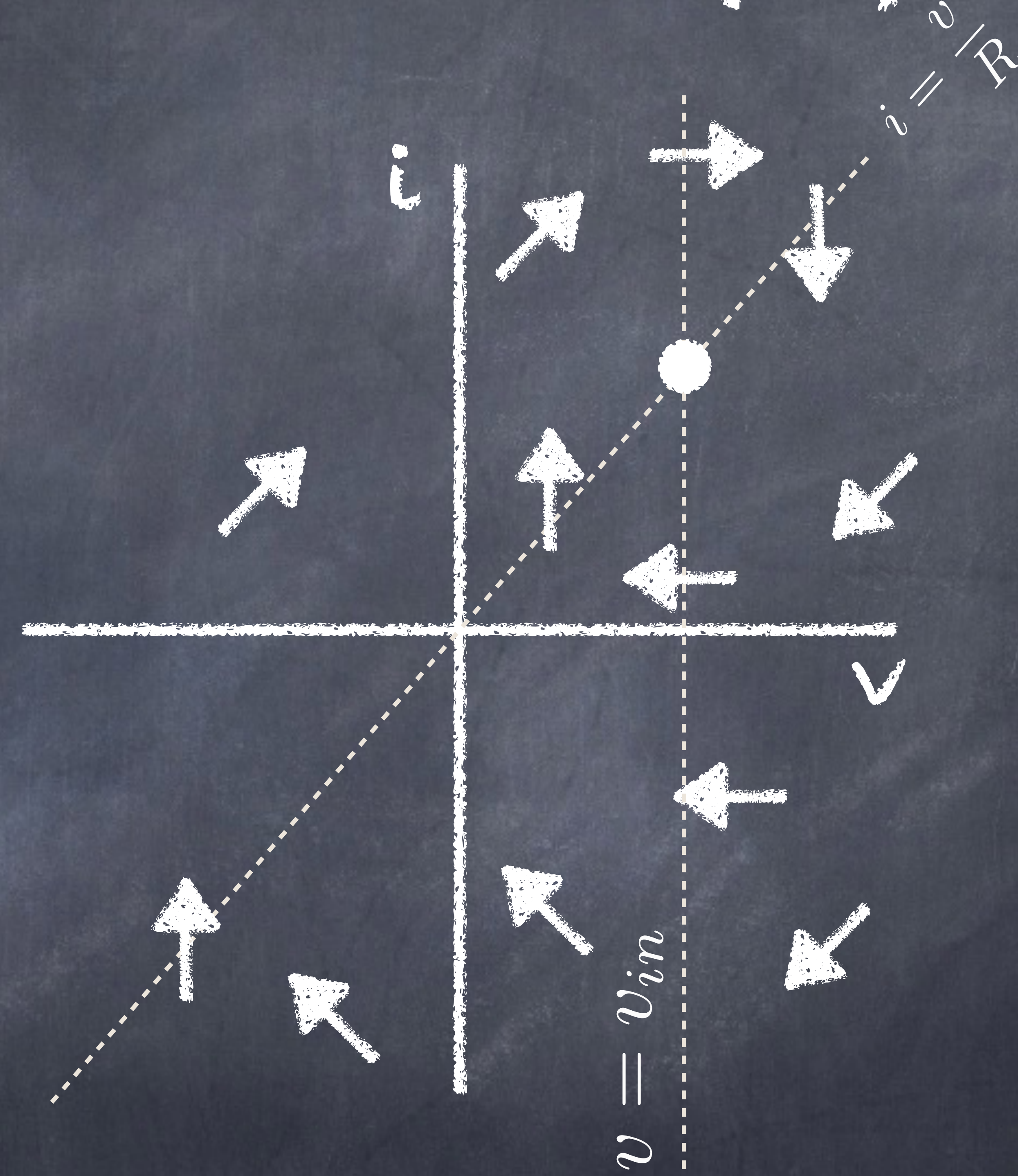
$$\frac{dv}{dt} = \frac{i}{C} - \frac{v}{RC}$$



When the switch is closed (on)

$$\frac{di}{dt} = \frac{v_{in} - v}{L}$$

$$\frac{dv}{dt} = \frac{i}{C} - \frac{v}{RC}$$



eigenvalues: $-\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$

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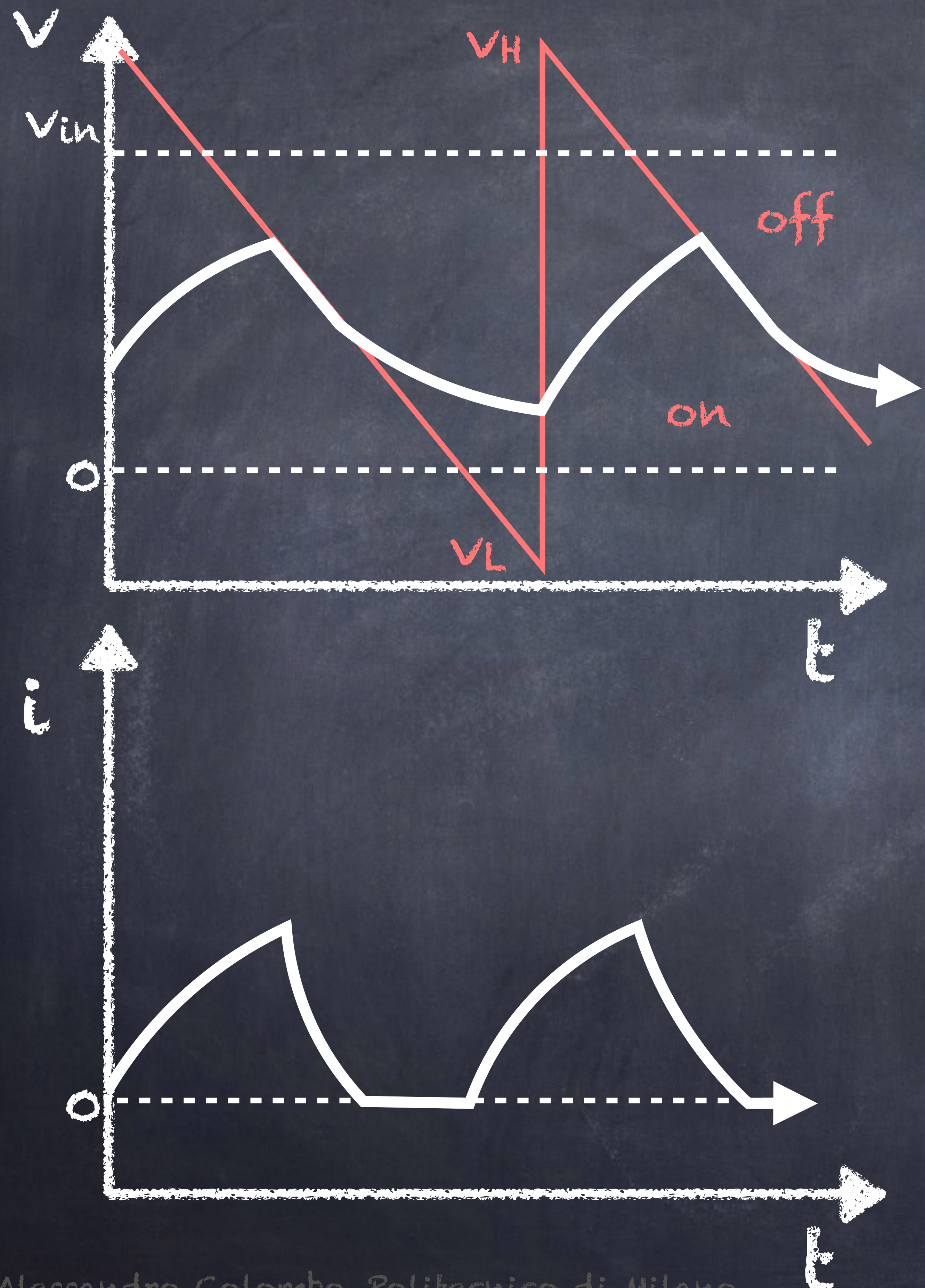


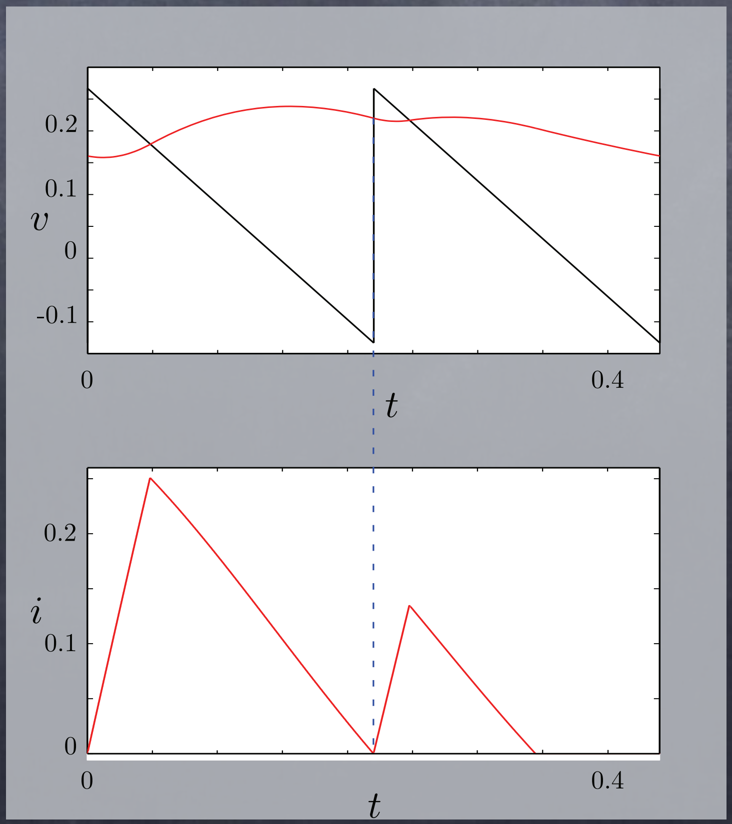
Boundary intersection crossing of the trivial (constant voltage) limit cycles:

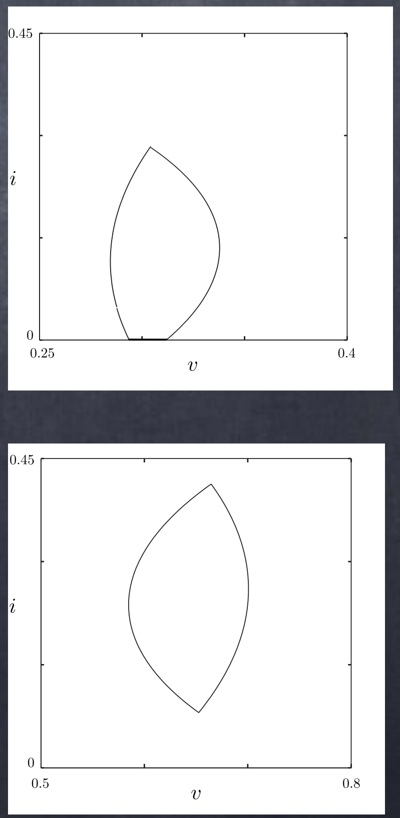
$$v_{in} = V_L$$

and

$$0 = V_H$$

Boundary intersection crossing transition CCD-DCD:





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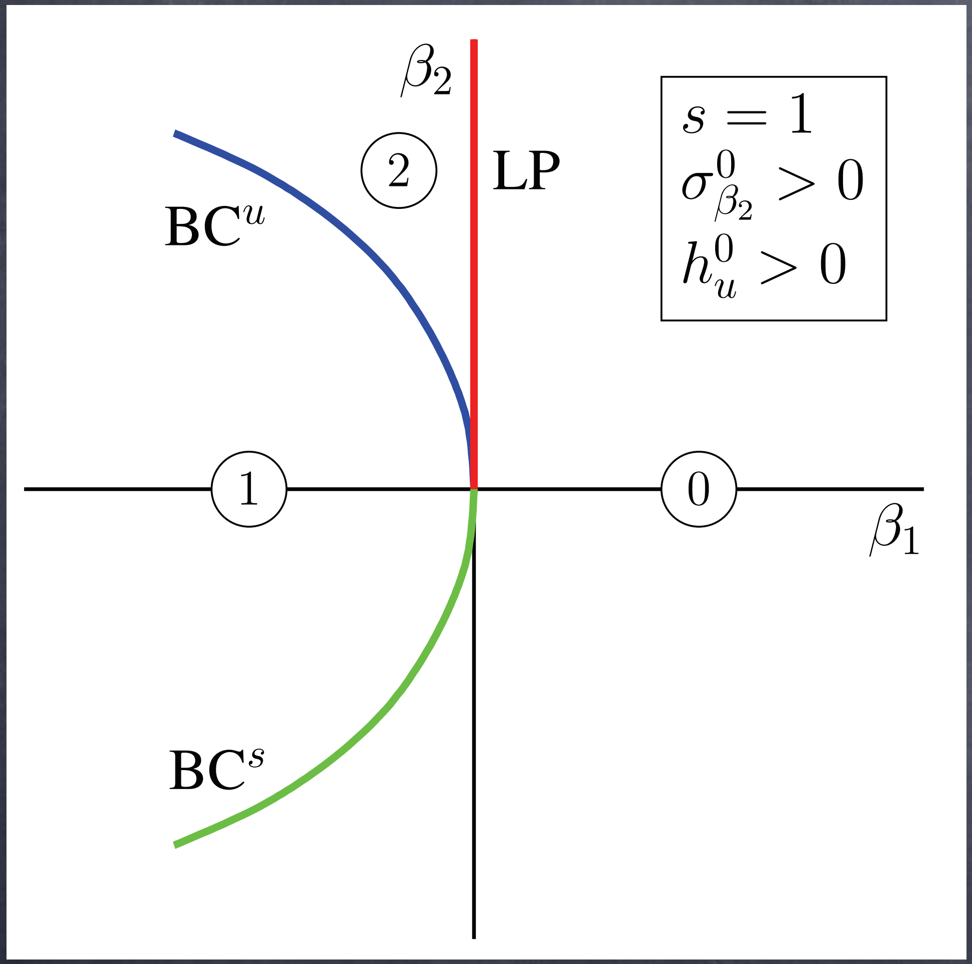
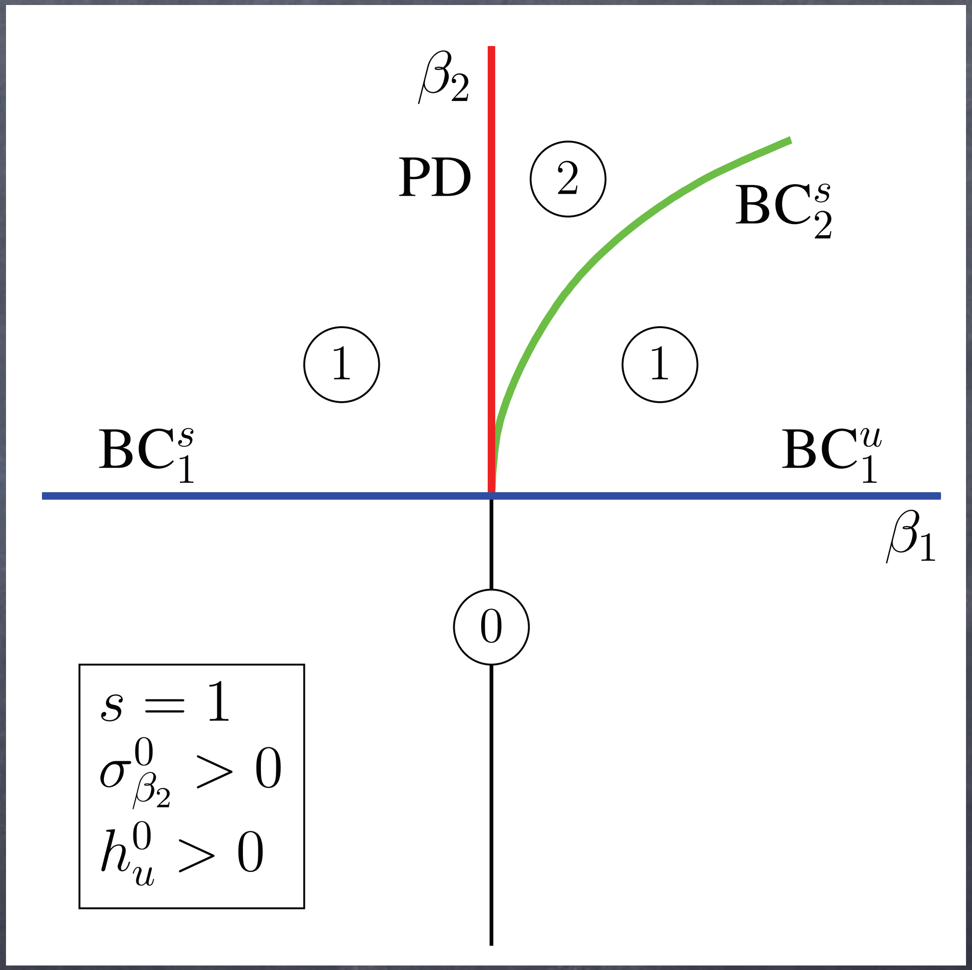
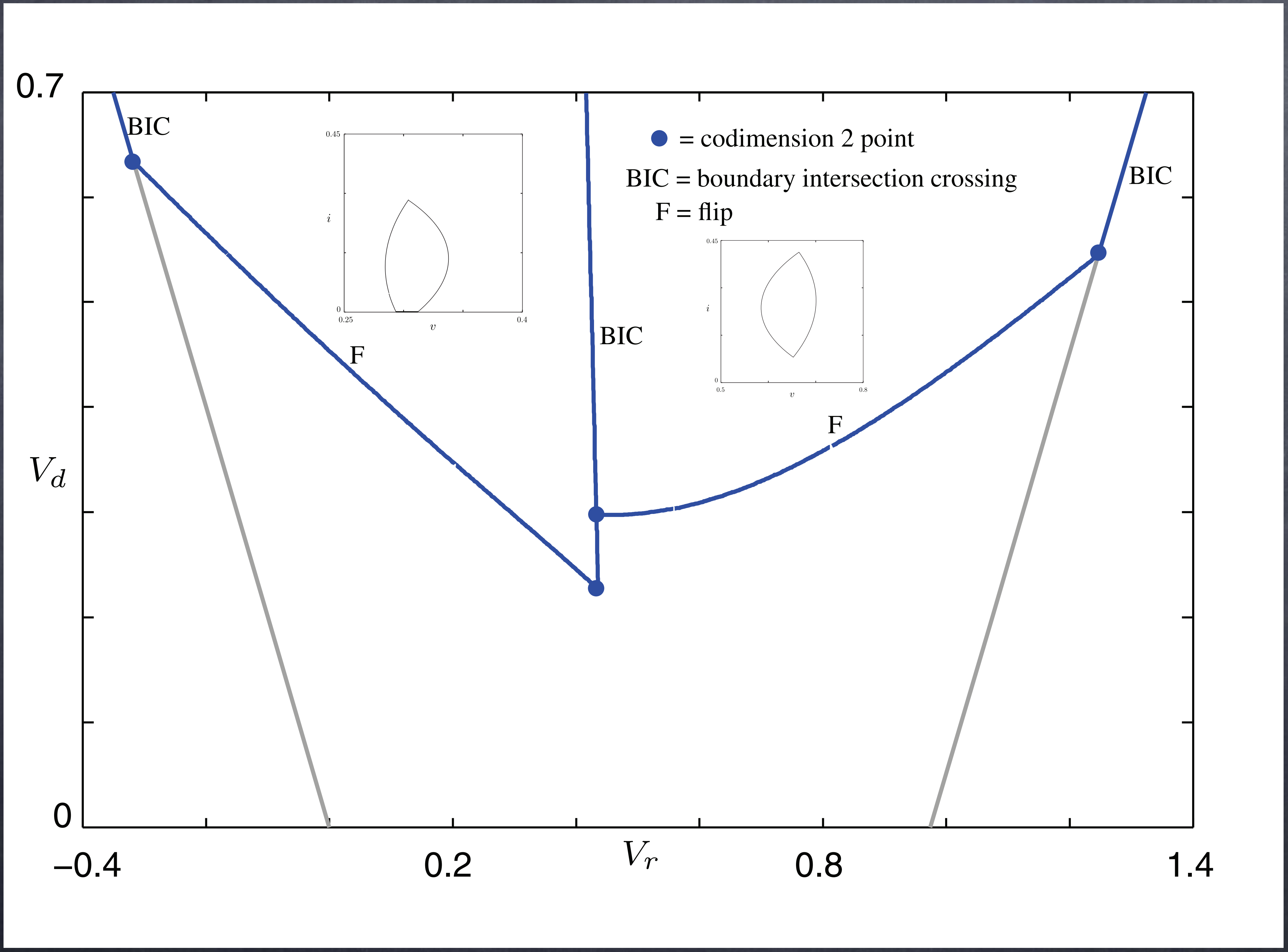
Flip/fold along a boundary intersection crossing






0.7
0
−0.4
0.2
0.8
1.4
V_d
V_r
BIC
F
BIC
BIC
F
= codimension 2 point
BIC = boundary intersection crossing
F = flip
0.45
i
0
0.25
v
0.4
0.45
i
0
0.5
v
0.8

Numerical BVP for the boundary intersection crossing:

$$\dot{x} = \phi(x, p_1, p_2)$$

$$x(T, p_1, p_2) - x(0, p_1, p_2) = 0$$

$$H_1(x(0), p_1, p_2) = 0, H_2(x(0), p_1, p_2) = 0$$

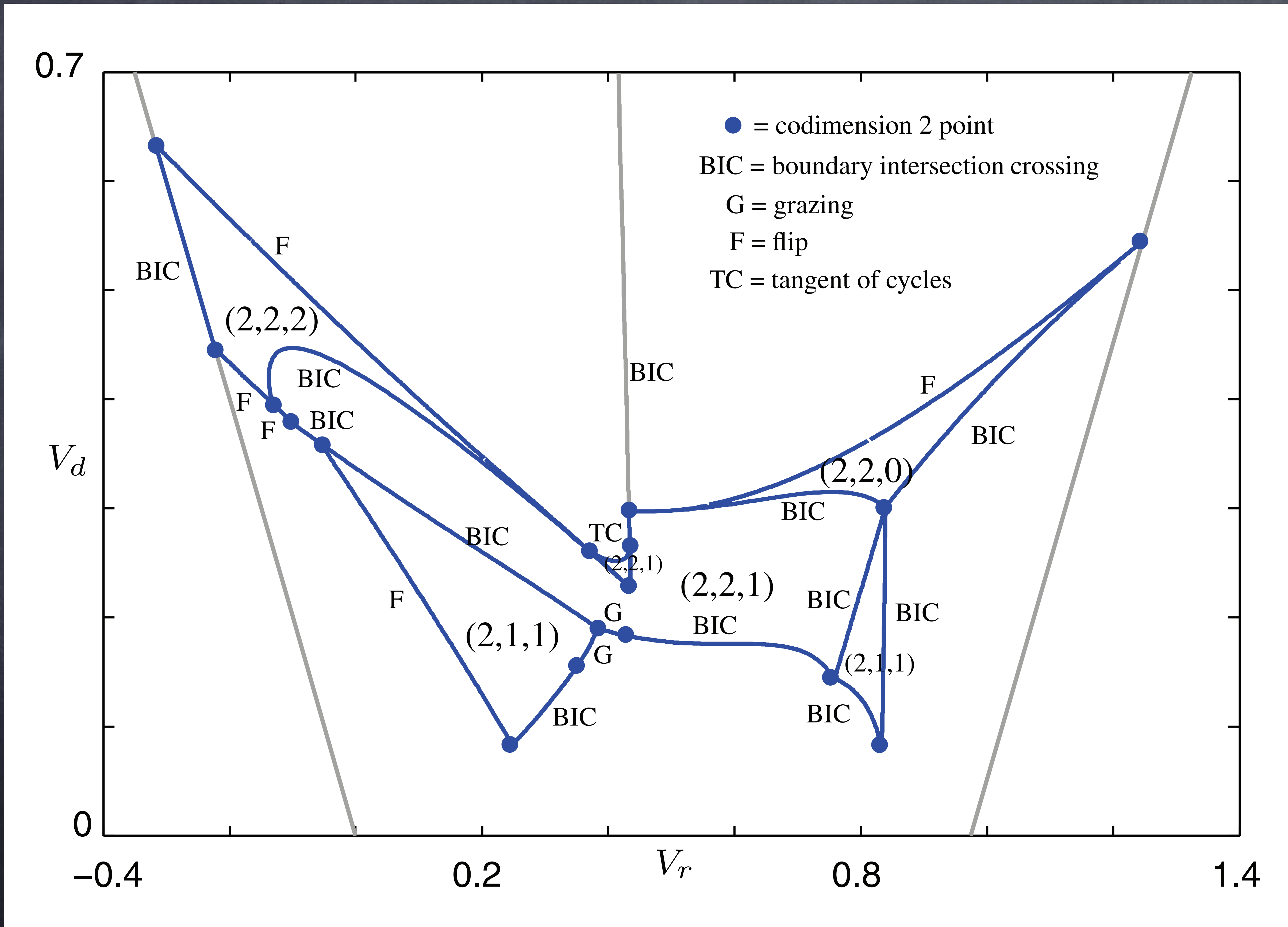
Numerical BVP for the grazing:

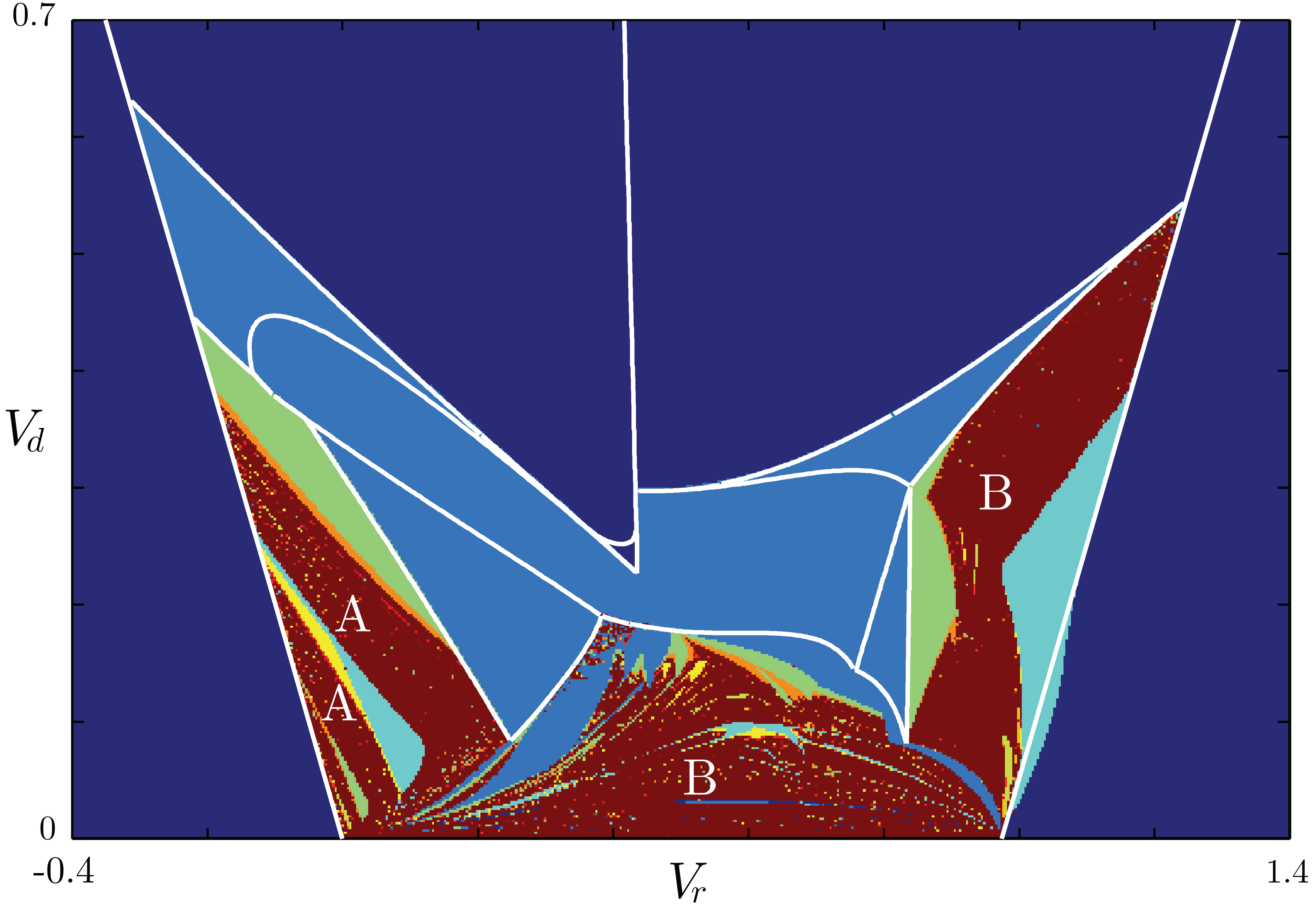
$$\dot{x} = \phi(x, p_1, p_2)$$

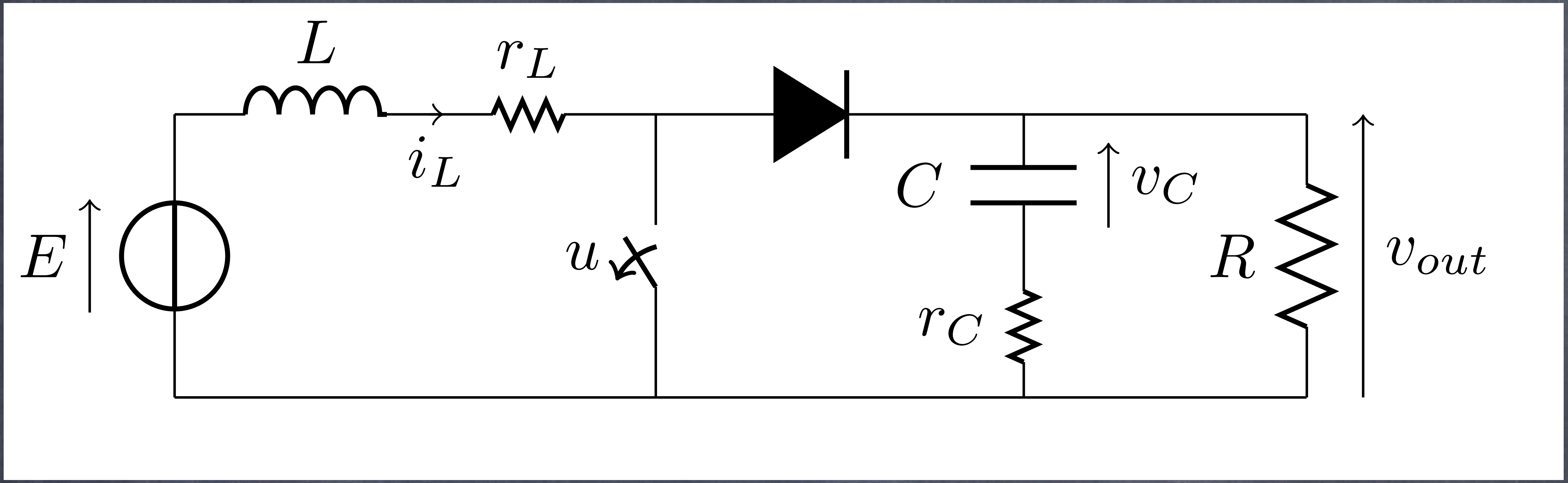
$$x(T, p_1, p_2) - x(0, p_1, p_2) = 0$$

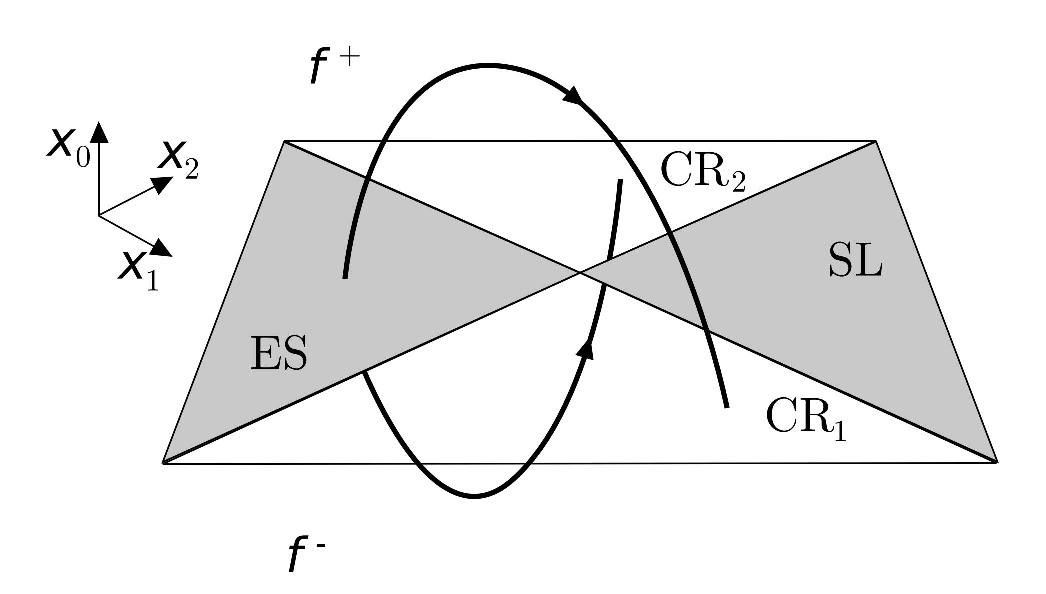
$$H(x(0), p_1, p_2) = 0$$

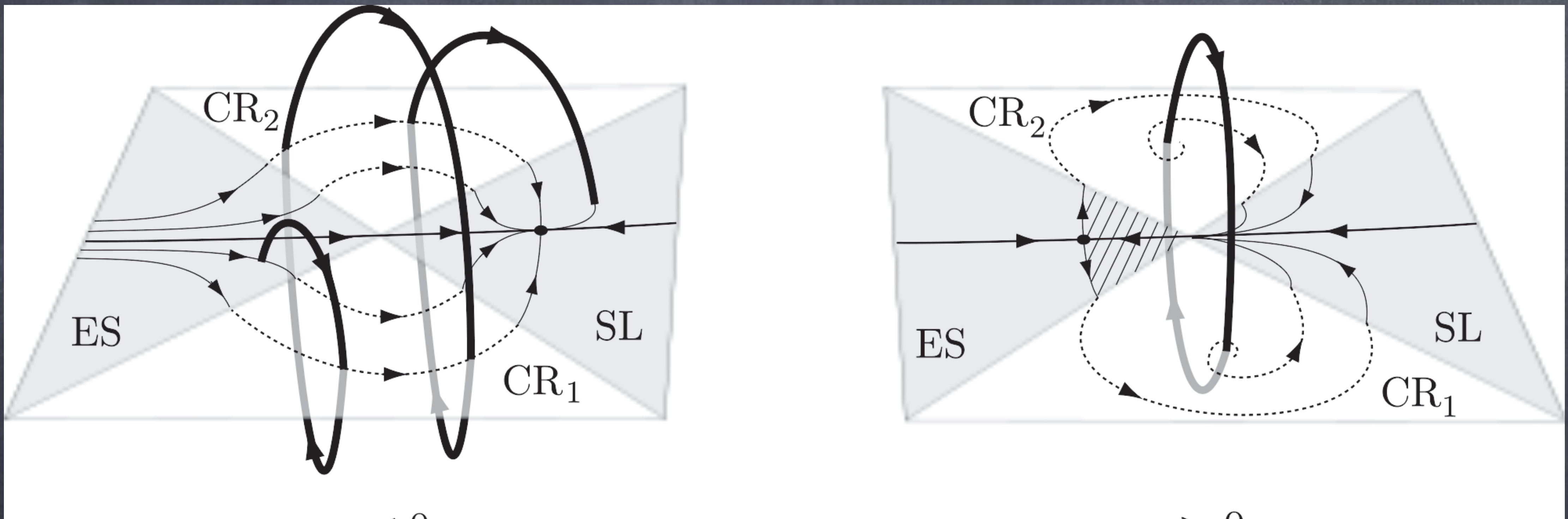
$$[f^{(off)}(x(0), p_1, p_2)]^T H_x(x(0), p_1, p_2)) = 0$$


= codimension 2 point
BIC = boundary intersection crossing
G = grazing
F = flip
TC = tangent of cycles
0.7
0
−0.4
0.2
0.8
1.4
V_d
V_r
BIC
F
(2,2,2)
BIC
F
F
BIC
BIC
F
(2,1,1)
G
G
BIC
TC
(2,2,1)
(2,2,1)
BIC
BIC
F
BIC
(2,2,0)
BIC
BIC
BIC
(2,1,1)
BIC


0.7
V_d
0
-0.4
V_r
1.4
A
A
B
B


L
r_L
i_L
E
u
C
v_C
r_C
R
v_out


f +
x0
x2
x1
CR2
SL
ES
CR1
f -



$p < 0$ $p > 0$

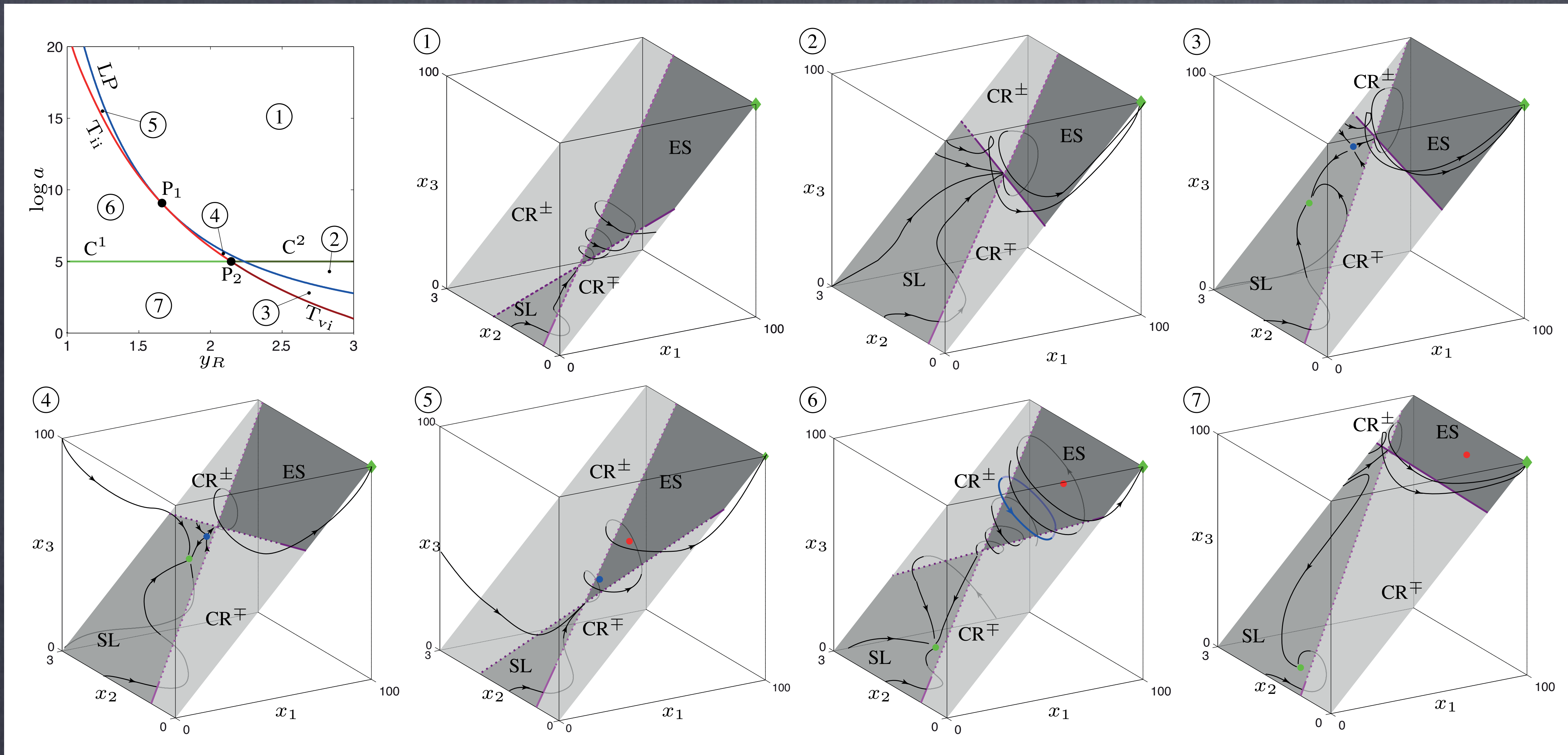


Figure 2: Bifurcation analysis of system (1). The bifurcation diagram in the upper left panel identifies 7 regions with topologically equivalent state portraits, depicted in panels ①–⑦. The bifurcation curves are: a limit point bifurcation (LP, blue), a two-fold bifurcation (light red (T_{ii}) regards an invisible-invisible two-fold, dark red (T_{vi}) regards a visible-invisible two-fold) and a cusp-fold bifurcation (light green (C^1) involves a crossing limit-cycle collapsing on the two-fold singularity, along the dark green (C^2) no cycle is observed). In the state portraits, light, medium, and dark gray identify crossing (($CR^{\pm}$) if trajectories cross from below to above the discontinuity surface, ($CR^{\mp}$) if trajectories cross from above to below), sliding (SL), and escaping (ES) regions of the discontinuity boundary. Light and dark purple lines are tangency points of the vector fields below and above, respectively. They are solid where the tangency is visible, dashed where the tangency is invisible. Diamonds identify standard equilibria, dots equilibria of a sliding or escaping vector-field. Green points are stable, blue points are saddles, red are unstable. Parameter values: $y_R = 2.5$, $a = 15$ (region 1), $y_R = 2.5$, $a = 4.5$ (2), $y_R = 2.5$, $a = 3.8$ (3), $y_R = 2$, $a = 6.2$ (4), $y_R = 1.2$, $a = 17$ (5), $y_R = 1.2$, $a = 10$ (6), $y_R = 1.2$, $a = 4.9$ (7). Other parameter values: $b = 0.01$, $w = 5$, $k = 30$.

Thank you