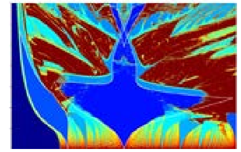
Dipartimento di Elettronica, Informazione e Bioingegneria

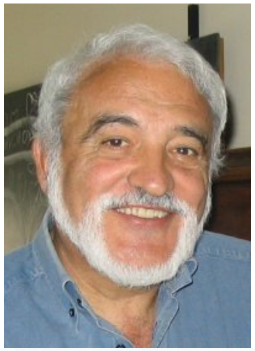
POLITECNICO DI MILANO

Bifurcation analysis of vehicle and vehicle+driver models

Fabio Della Rossa

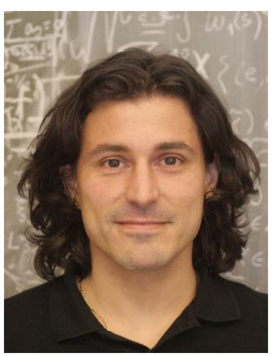


Who we are…



Sergio Rinaldi
Em. prof.

Carlo Piccardi
Full prof.



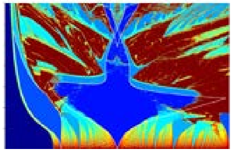
Fabio Dercole
Ass. prof.

Alessandra Gragnani

Massimo Miari

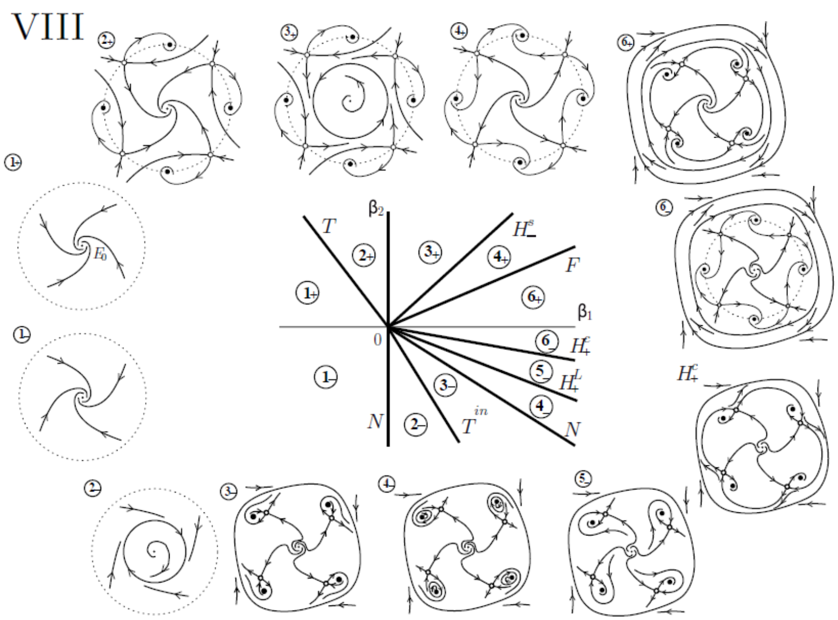
Alessandro Colombo

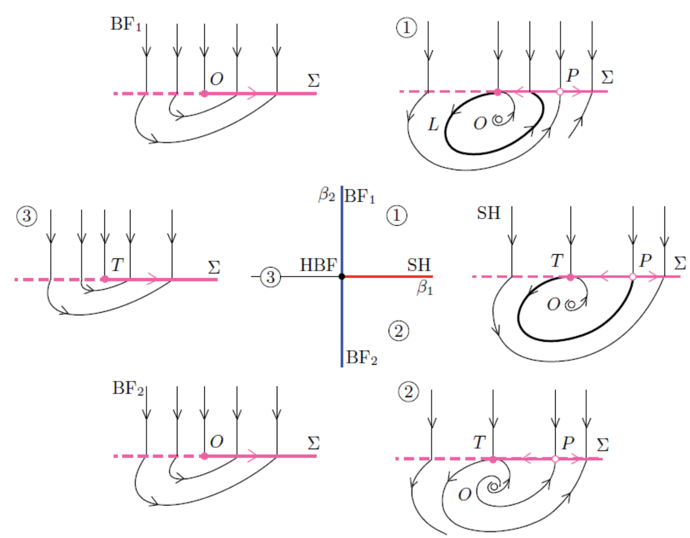
Fabio Della Rossa

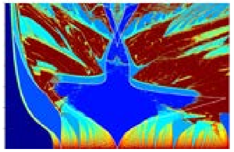


Theory

✓ Bifurcation studies in smooth and nonsmooth systems







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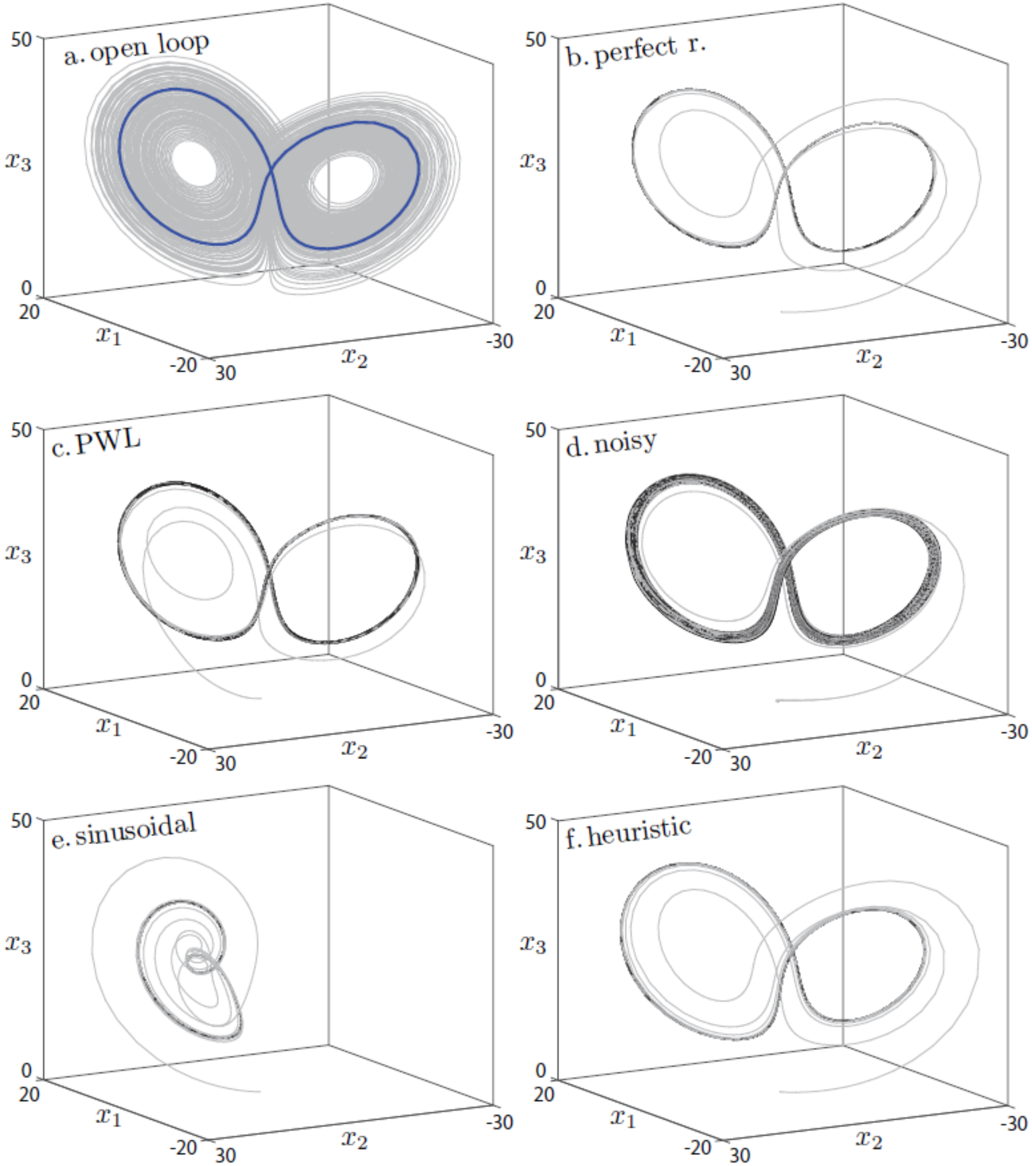
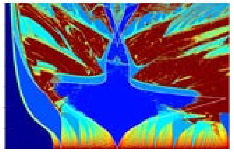
- ✓ Bifurcation studies in smooth and nonsmooth systems
- ✓ Nonlinear control





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- ✓ Bifurcation studies in smooth and nonsmooth systems
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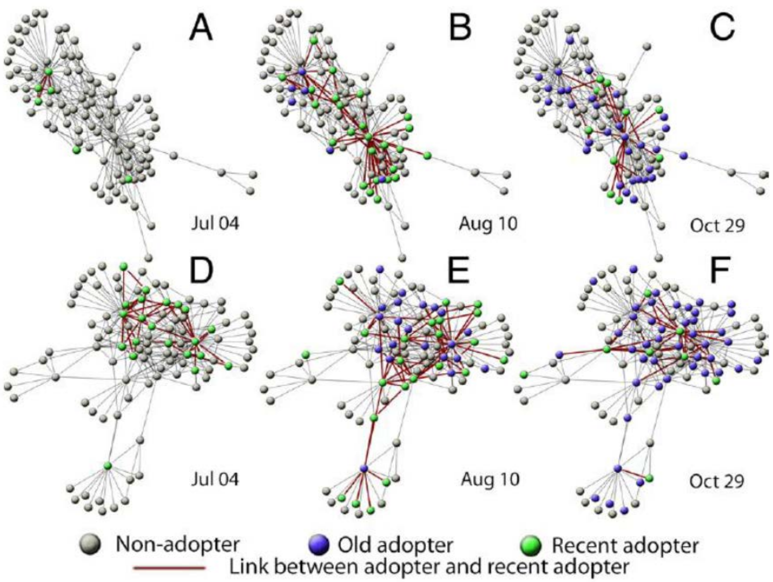
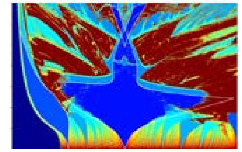
- ✓ Complex networks analysis





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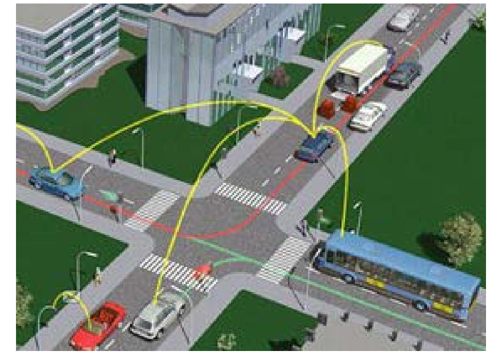
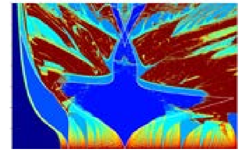


- ✓ Bifurcation studies in smooth and nonsmooth systems
- ✓ Nonlinear control
- ✓ Complex networks analysis
- ✓ Intelligent transportation systems



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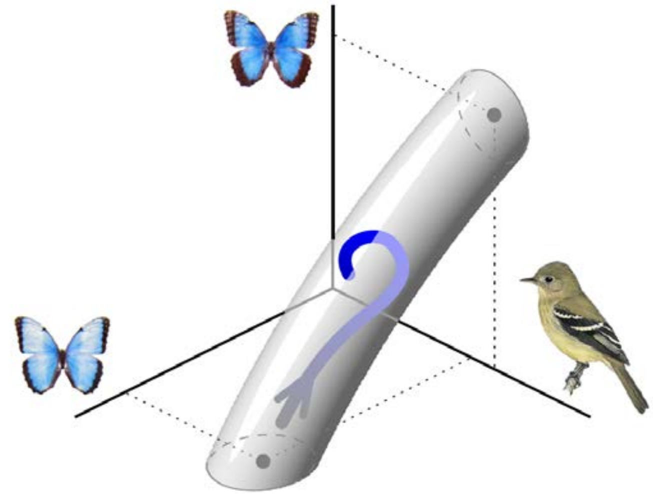

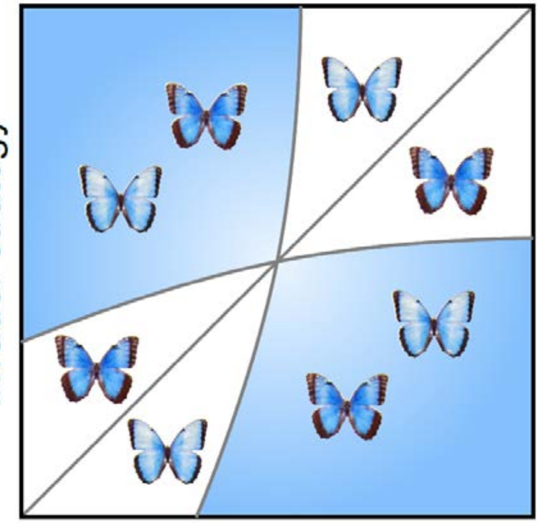

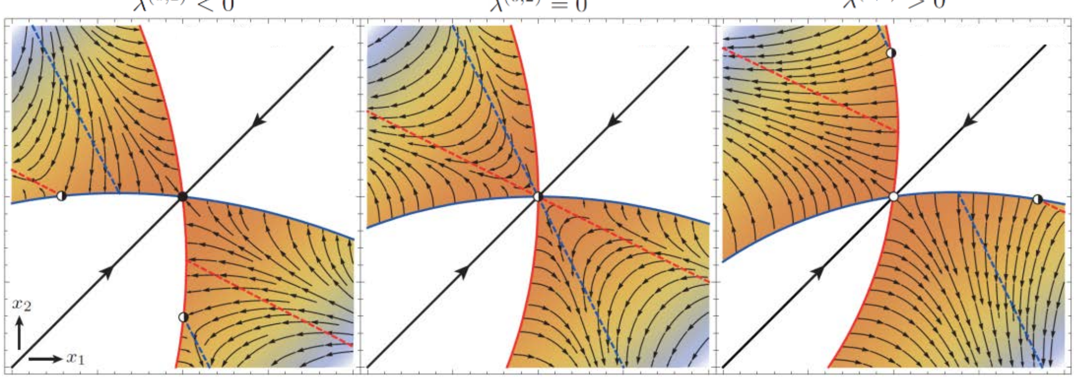

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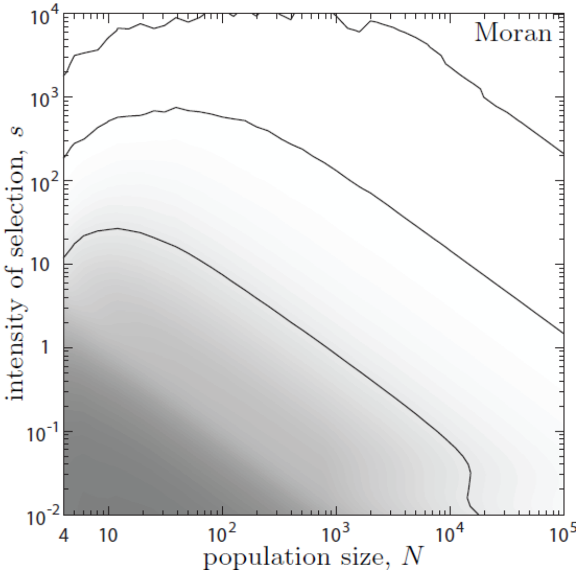
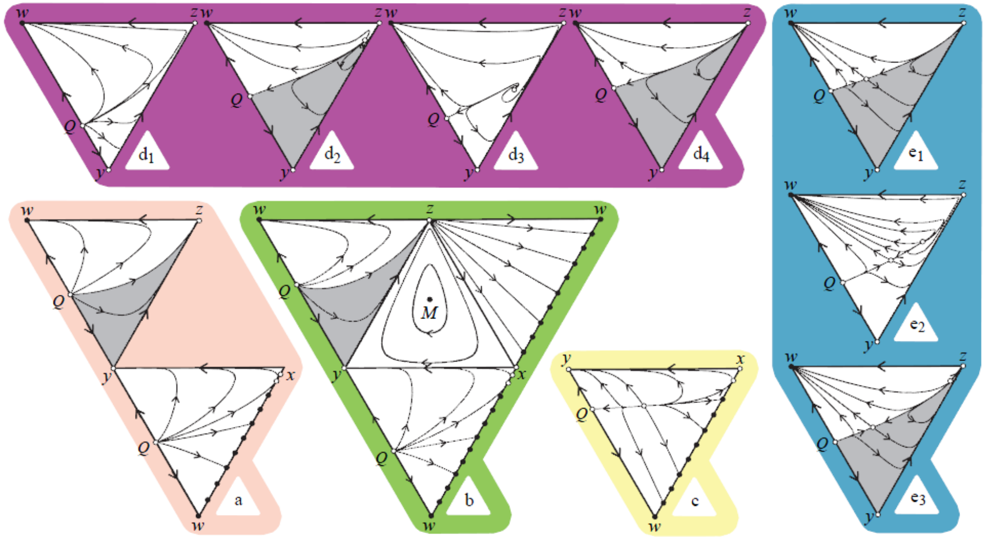
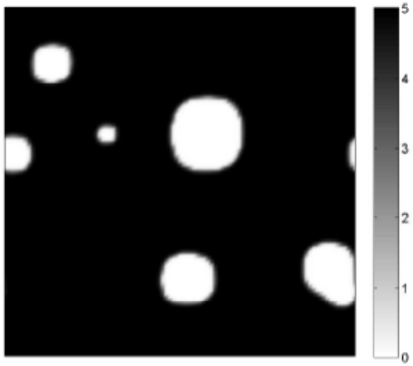
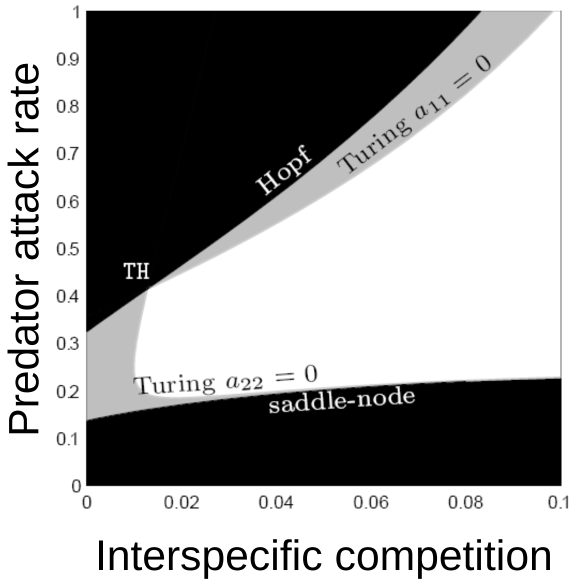


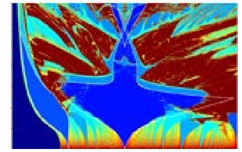


Applications





- ✓ Stability of homogeneus solutions in spatial distributed ecological models



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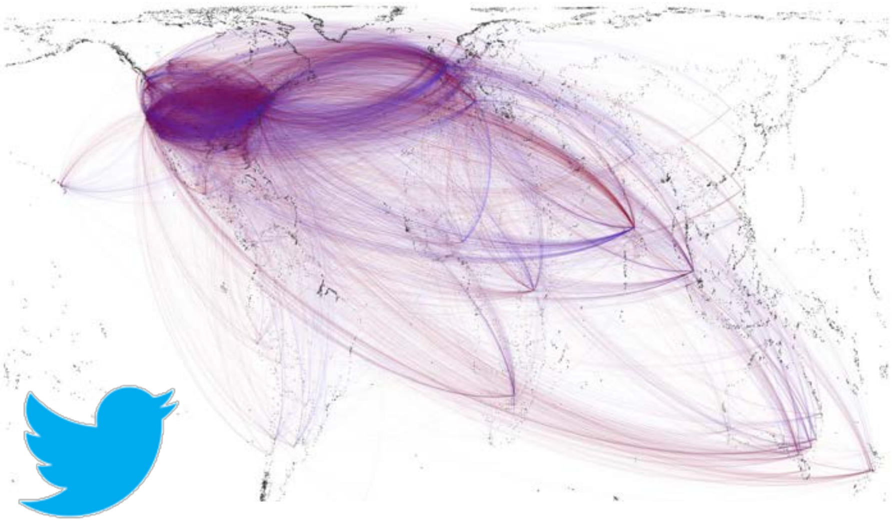

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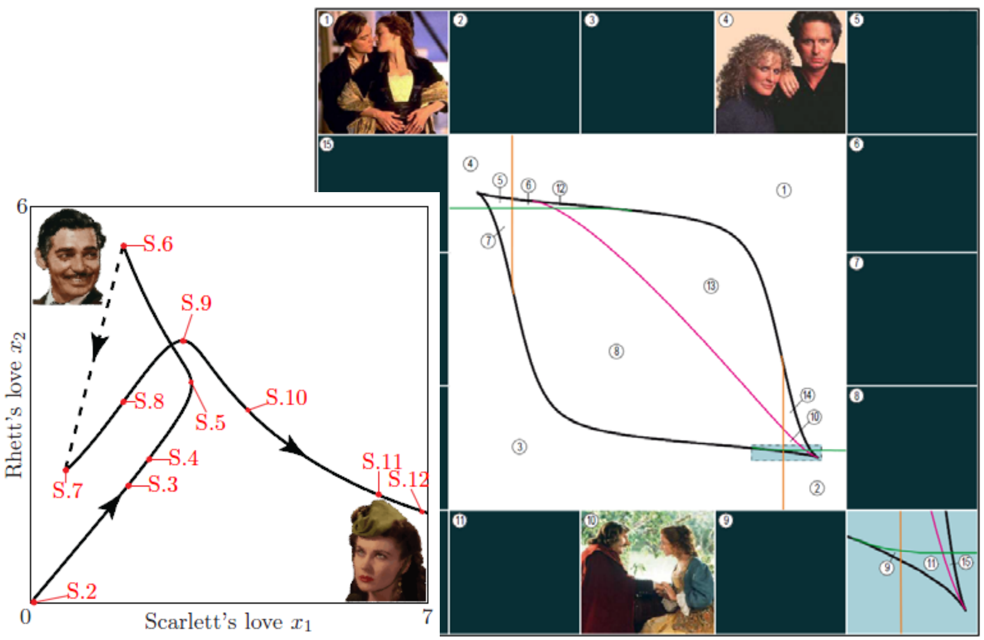

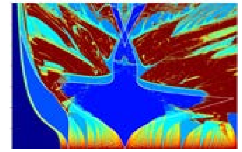

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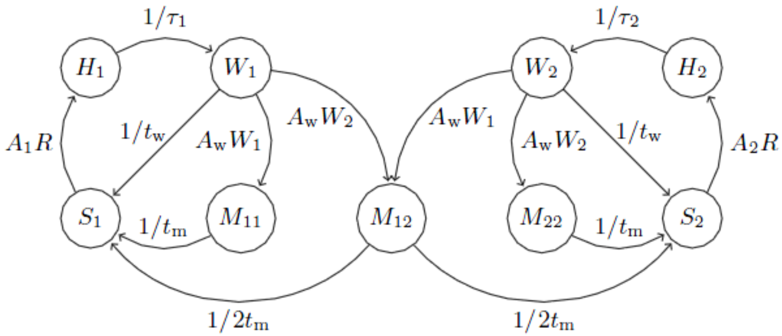

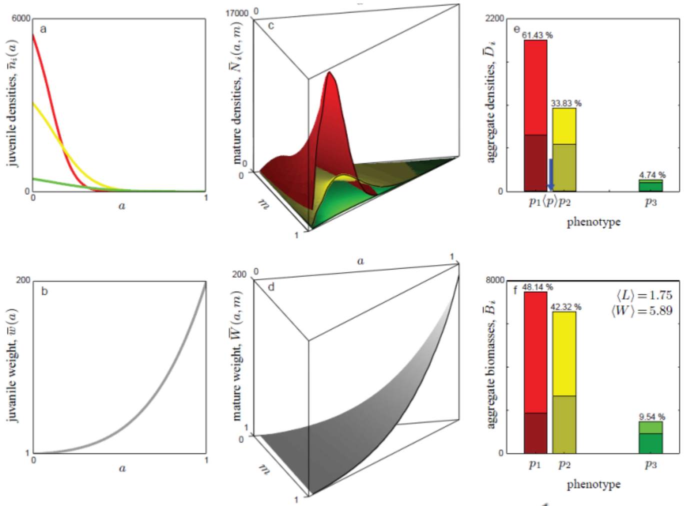

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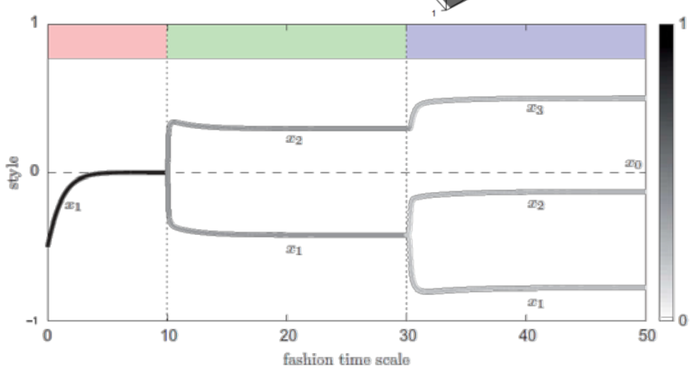

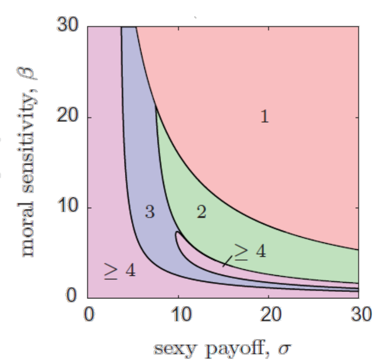

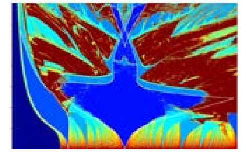

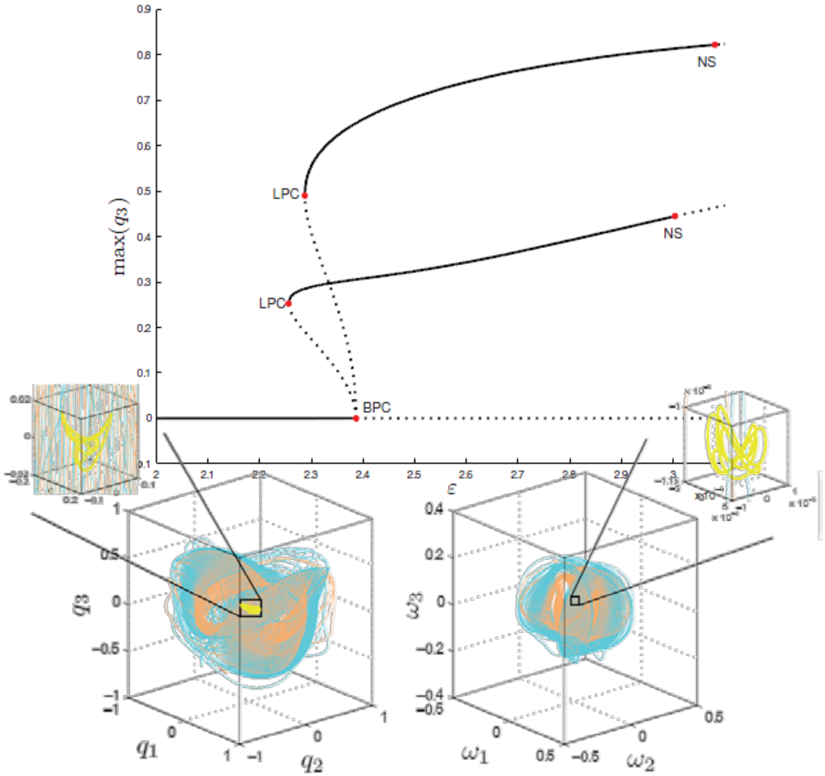

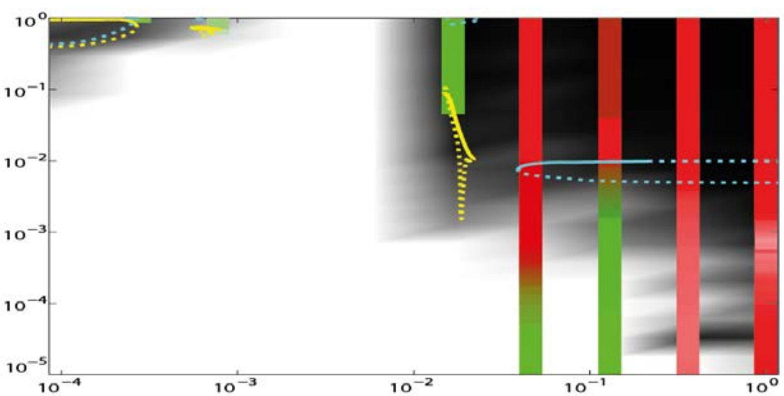

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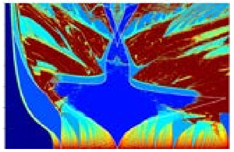

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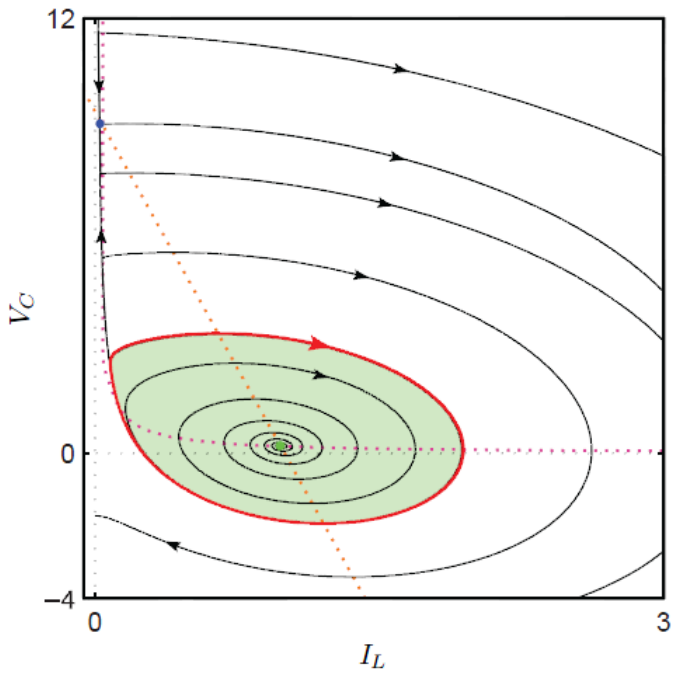

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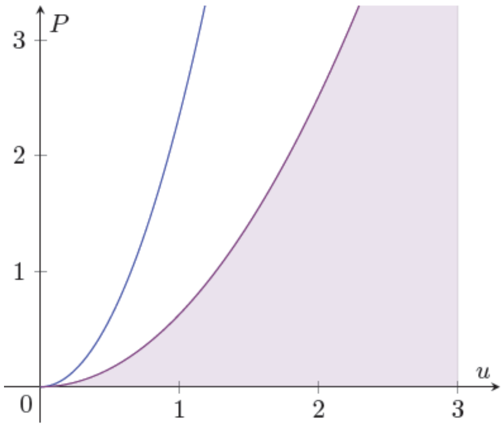

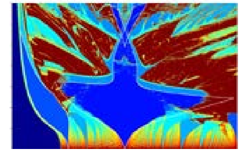

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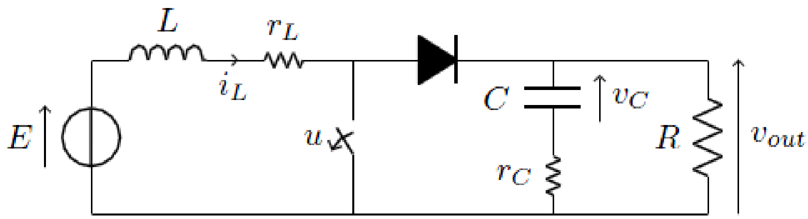
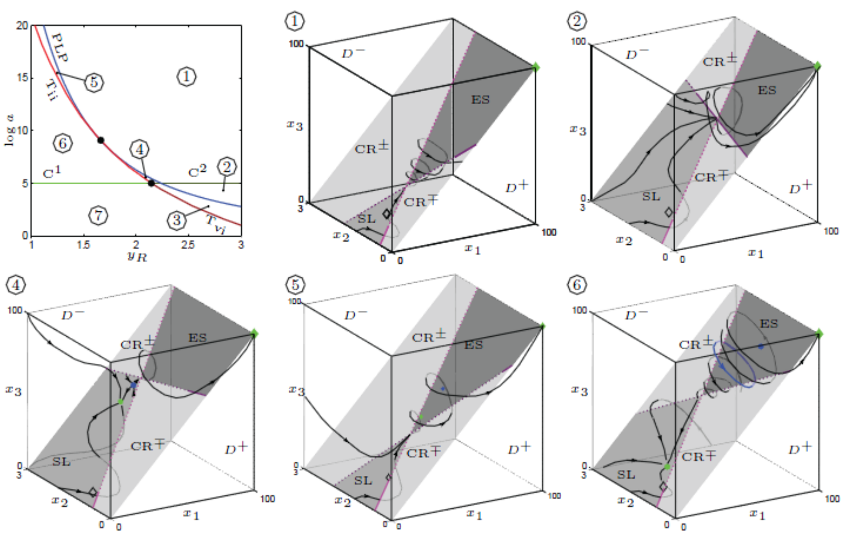


Figure 1: DC-DC boost converter.



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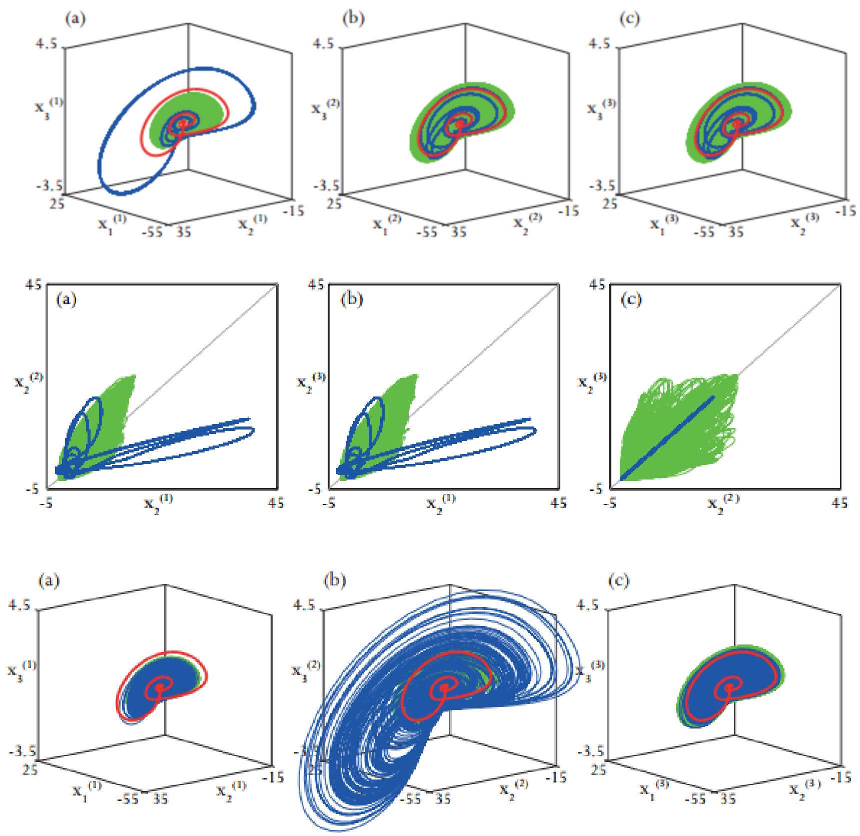

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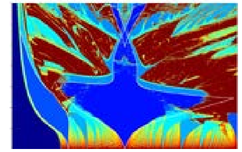

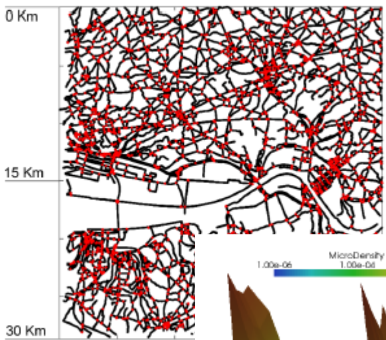

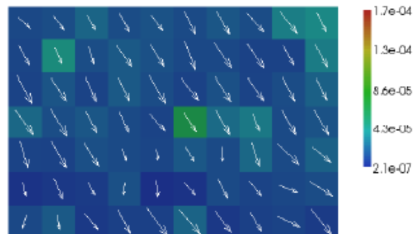

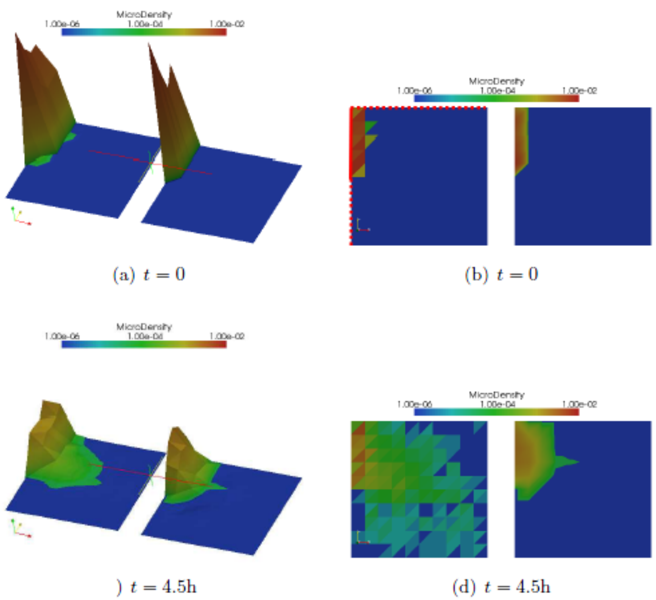

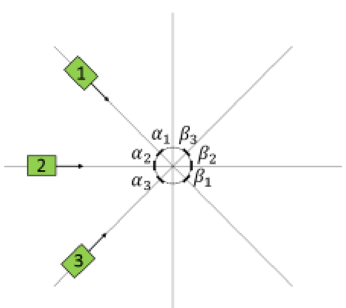

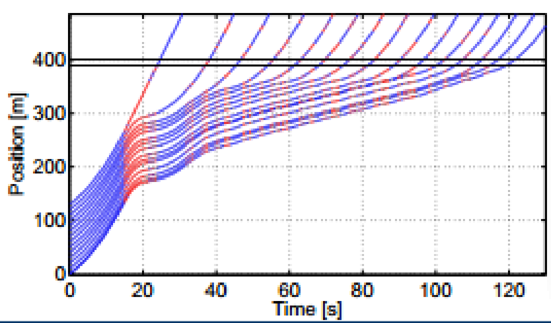

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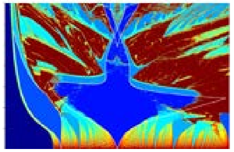
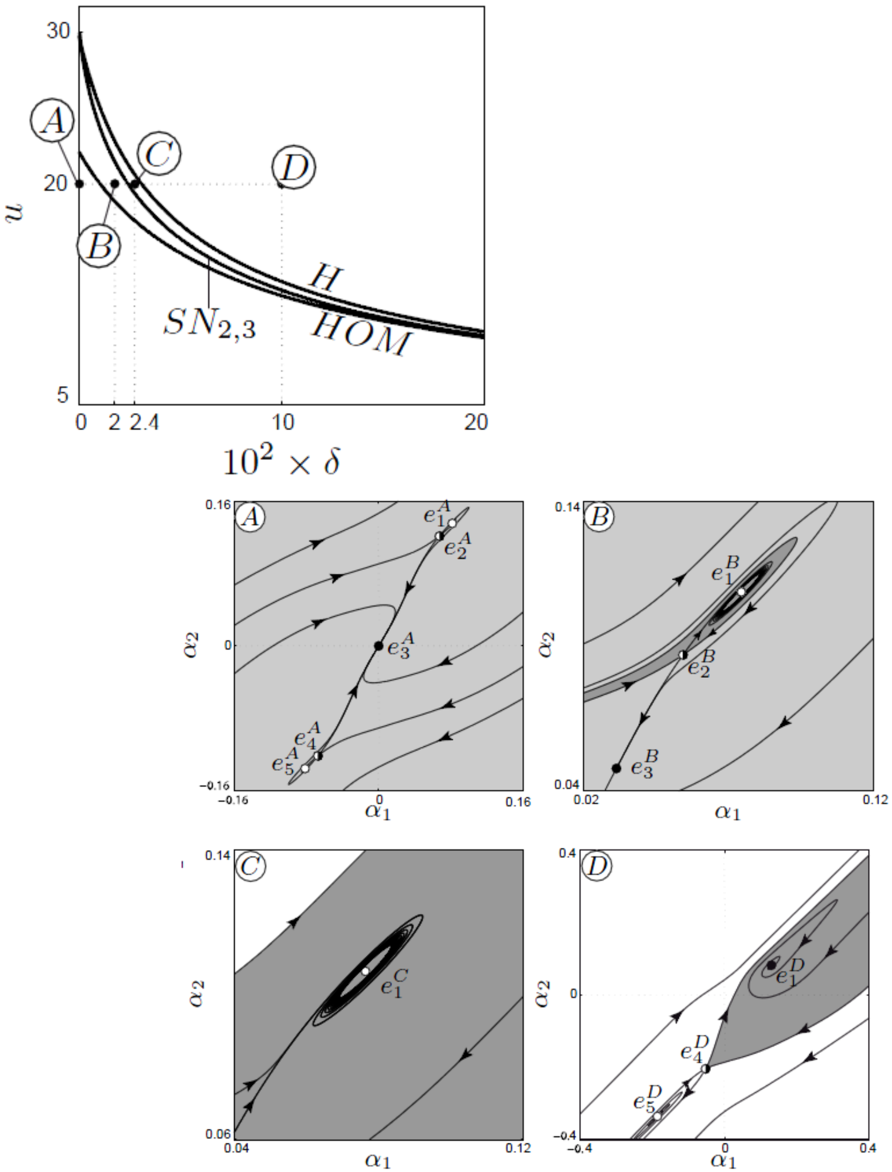
- ✓ Analysis and control of vehicular traffic





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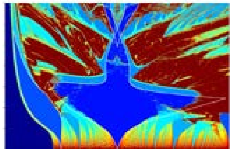

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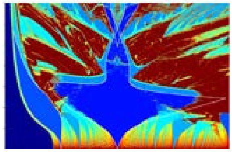

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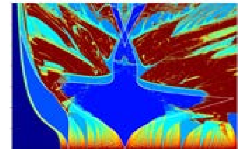
THE AUTOMOBILE MODEL AND ITS BA

- The single track model
- Model validation
- Bifurcation analysis of the vehicle model
- Understeering vs. Oversteering vehicles

THE HUMAN-IN-THE-LOOP SYSTEM

- Driver models and vehicle-driver coupling
- Model validation
- Bifurcation analysis of the vehicle and driver model
- Results comparison
- An extension and further research directions



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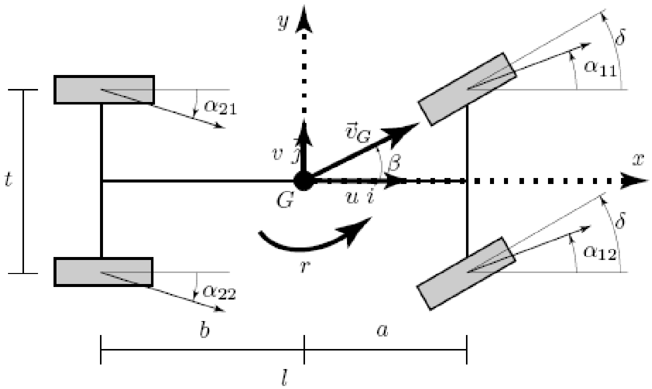
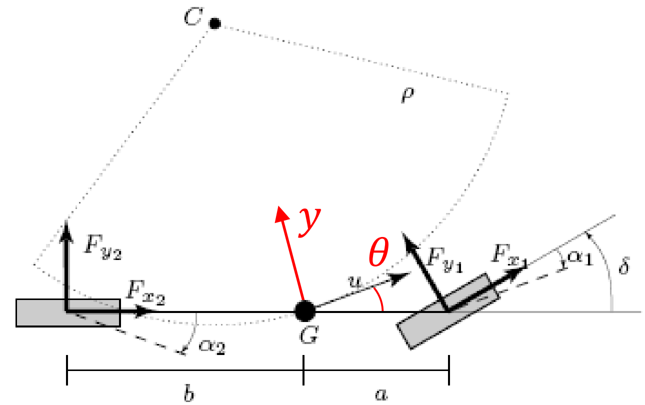
The single track model





SIMPLIFYING HYPOTHESES

- the forward speed *u* is constant;
- the centre of gravity lies at the ground level;
- the vehicle body is modelled referring to its longitudinal axis;
- the resultant of the forces acting at the front and rear axles are applied at the centres of the axles;
- the slip angles *αi*, *i* = 1, 2, and the steering angle *δ* (Figure 1) are small and
- no longitudinal forces are acting at the wheels.

$$m\,(\ddot{y} + u\,\dot{\theta}) = F_{y_1}(\alpha_1) + F_{y_2}(\alpha_2)$$

$$I_z\,\ddot{\theta} = F_{y_1}(\alpha_1)a - F_{y_2}(\alpha_2)b$$

Where the rear and front slip angles can be obtained as:

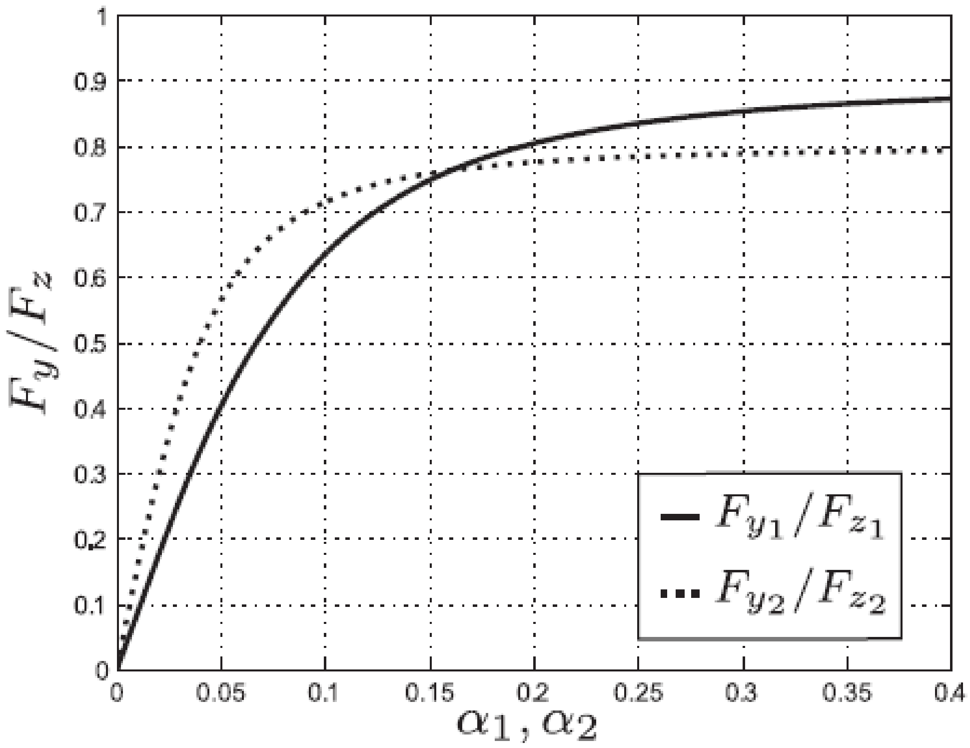
$$\alpha_2 = -\frac{\dot{y} - b\dot{\theta}}{u}, \qquad \delta - \alpha_1 = \frac{\dot{y} + a\,\dot{\theta}}{u}$$

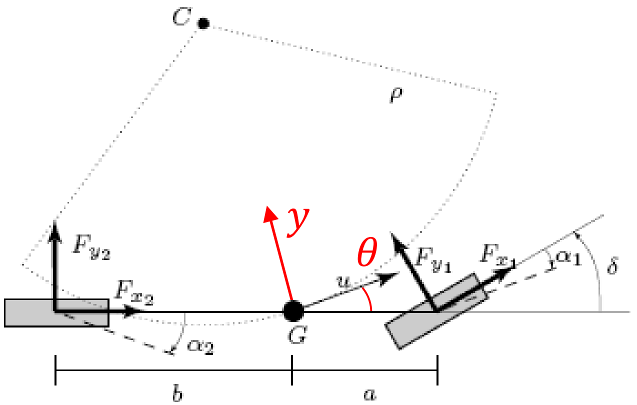
2 D.O.F. MODEL IN $\dot{y}$ and $\dot{\theta}$

The single track model – non-linearities

Tyre characteristics

UN$_1$ vehicle





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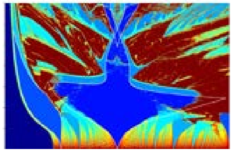
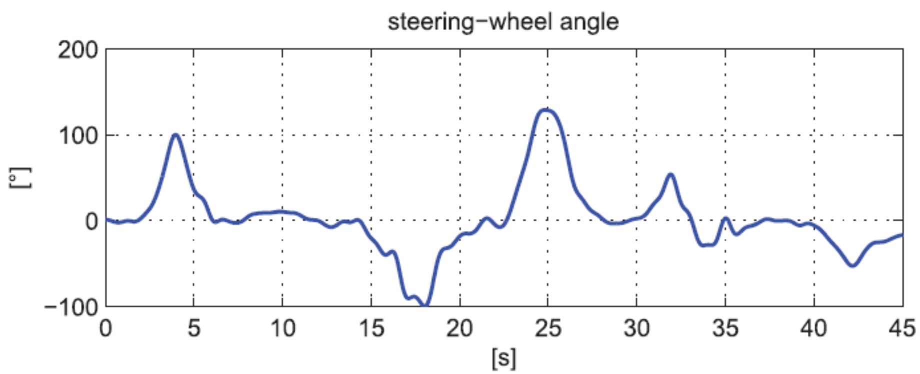
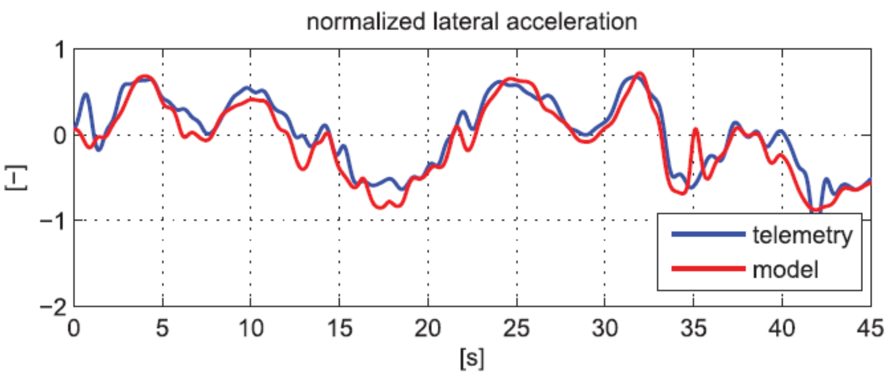
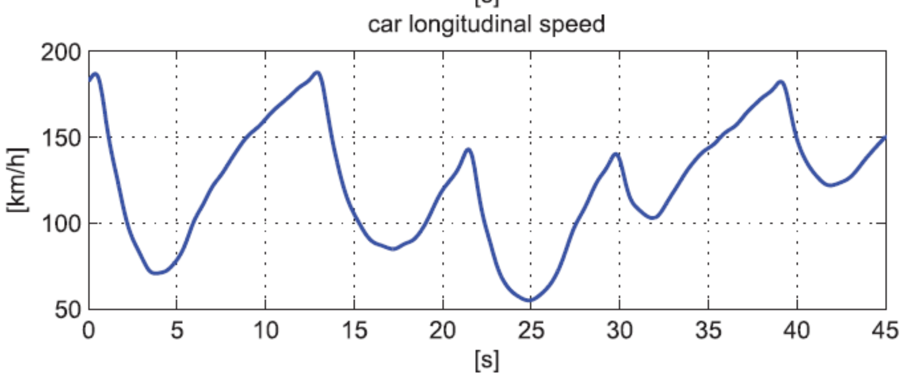


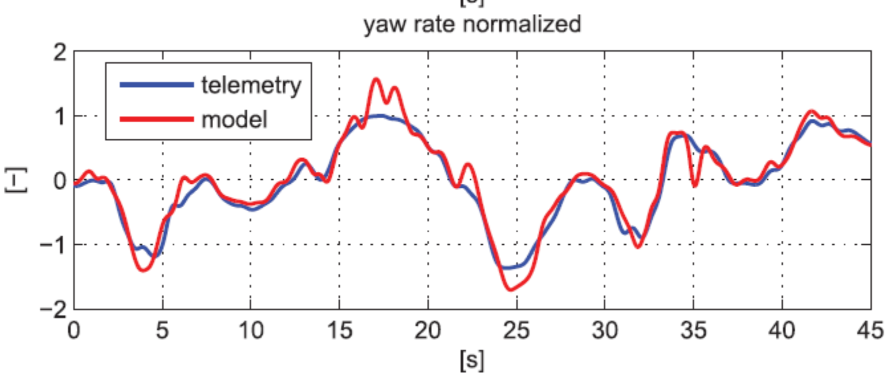
Model validation

	Value	*Unit*
Vehicle mass (m)	550	kg
Vehicle yaw moment of inertia (I_z)	750	kg m^2
Distance from front axle to c.g. (a)	1.64	m
Distance from rear axle to c.g. (b)	0.98	m



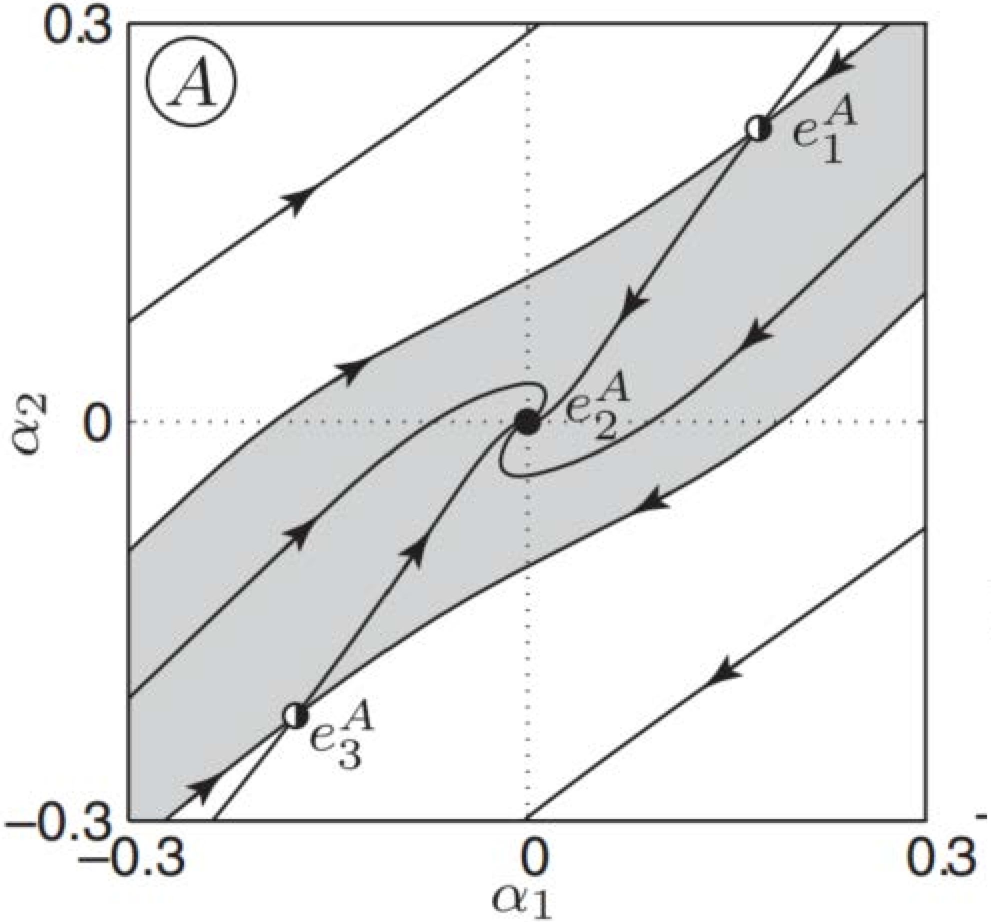


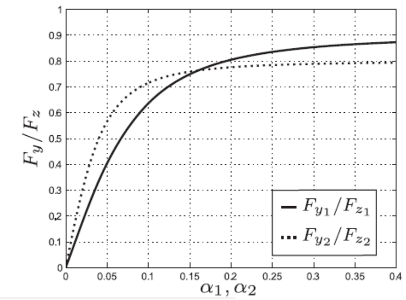


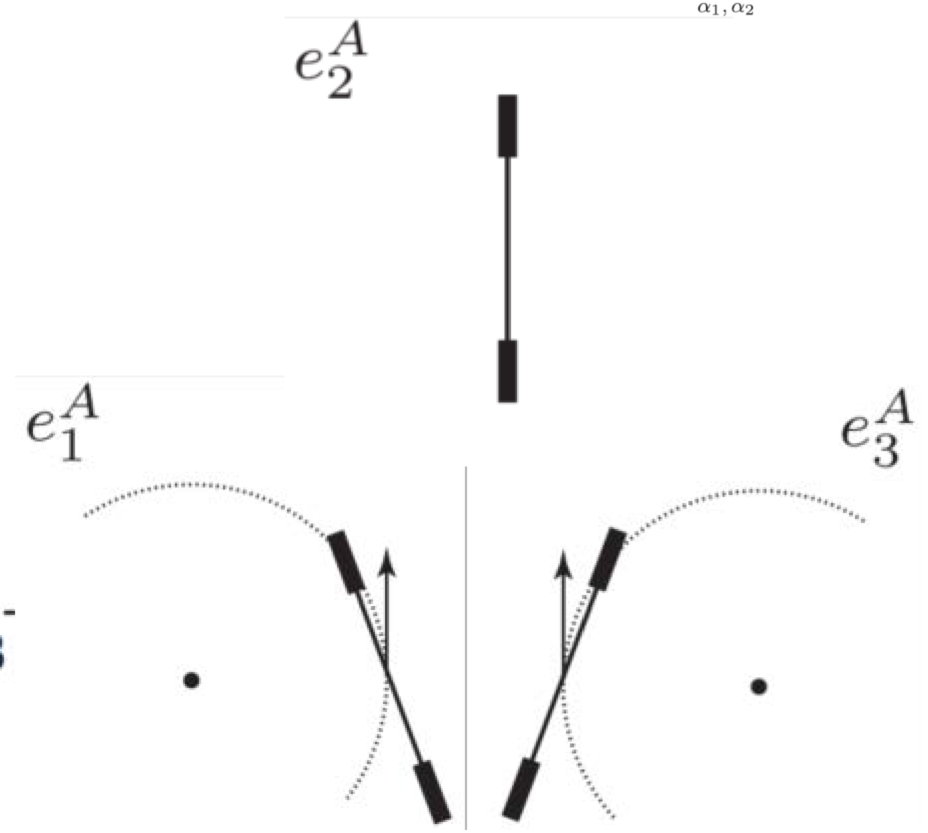


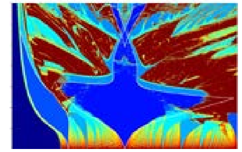
Try it with PPlane

The single track model – the UN_1 case



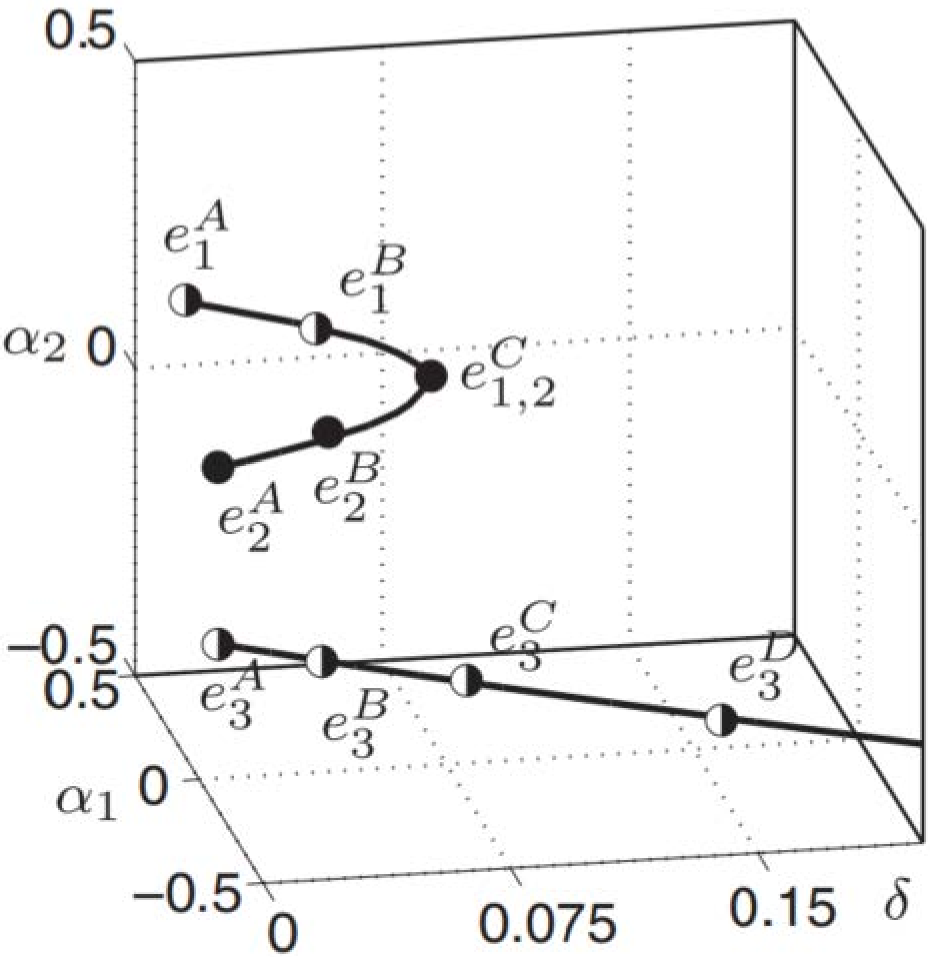


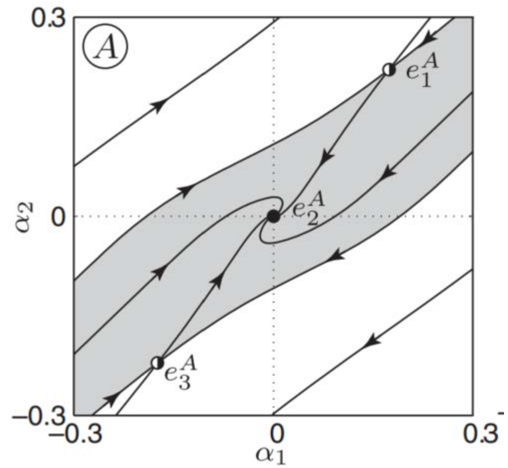


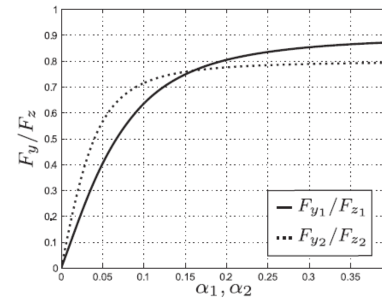


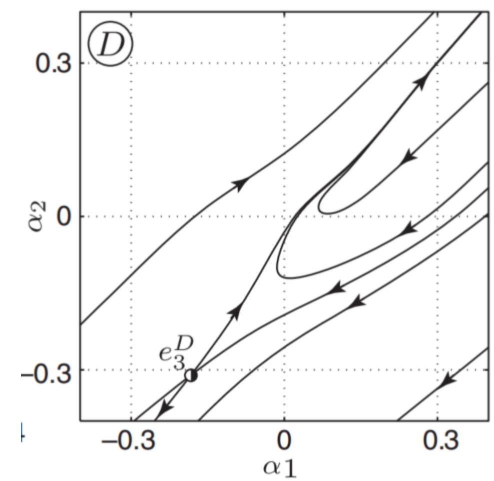
Rinaldi, Piccardi, Dercole, Gragnani, Miari, Colombo, Della Rossa

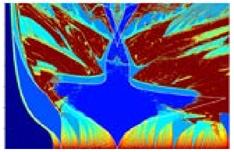
The single track model – the UN_1 case











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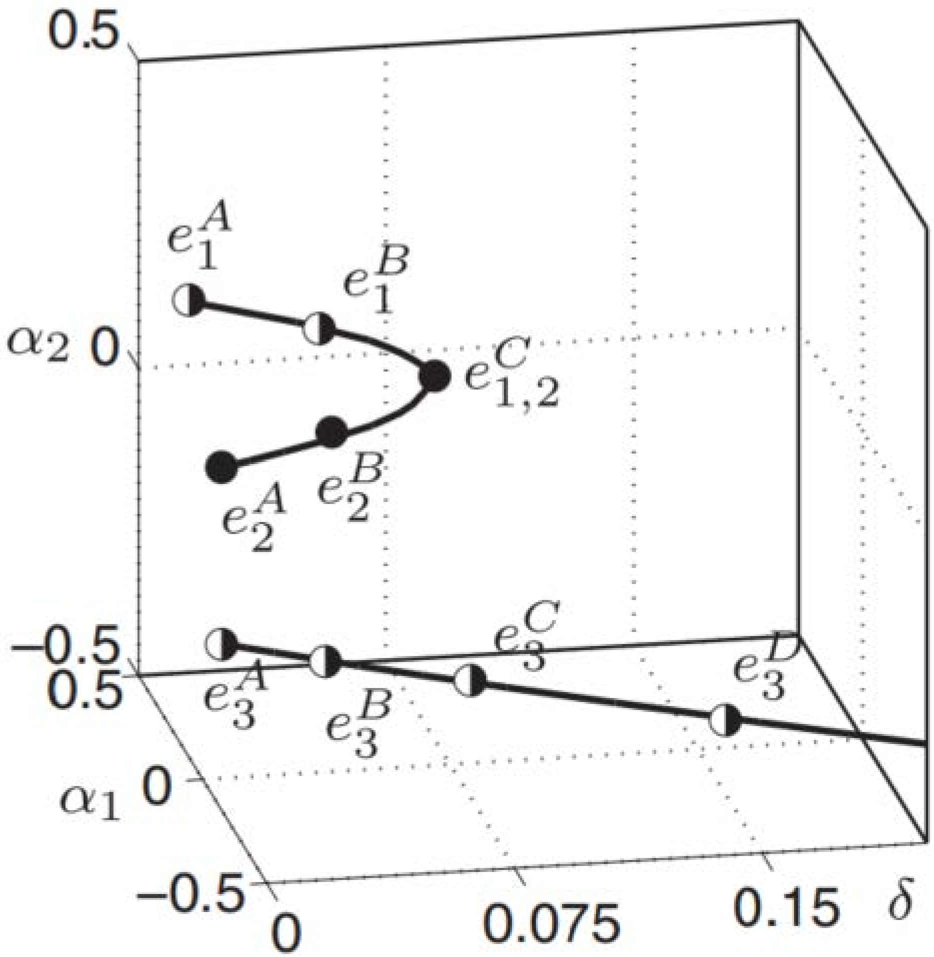
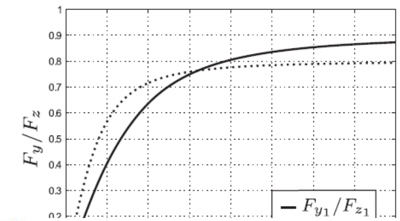
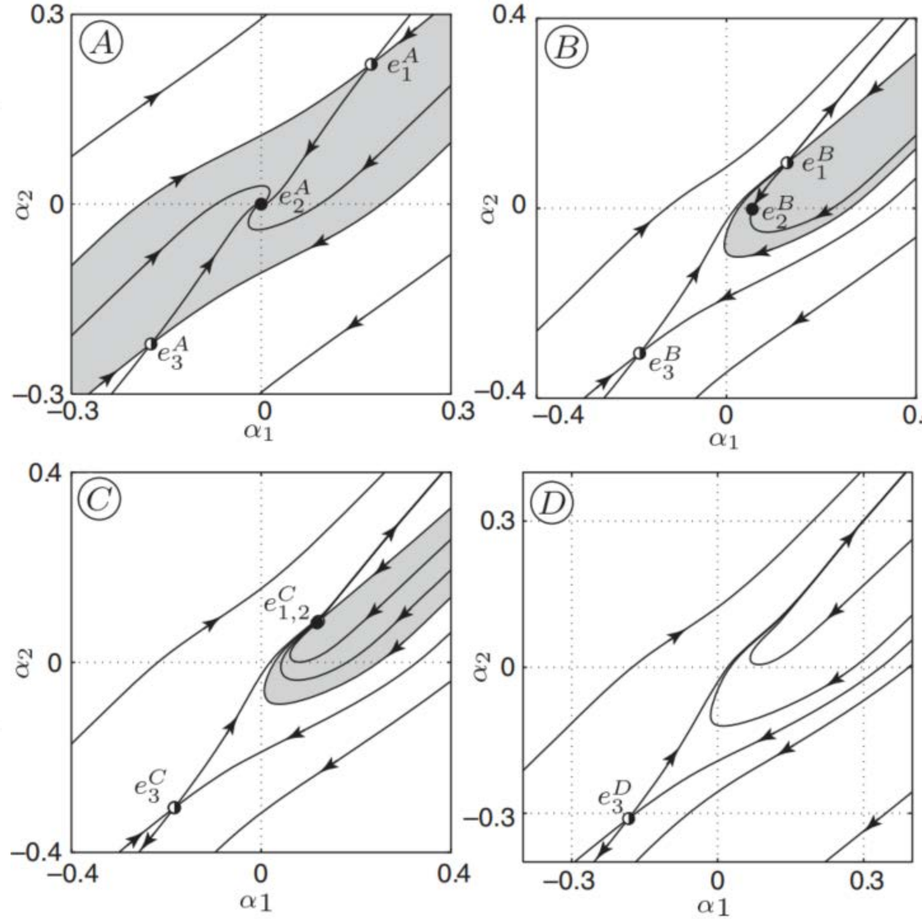
Miari, Colombo, Della Rossa

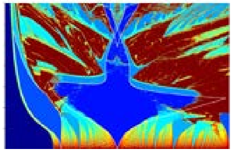
Try it with PPlane

Bifurcation analysis – the UN_1 case









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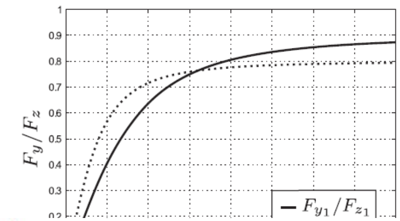
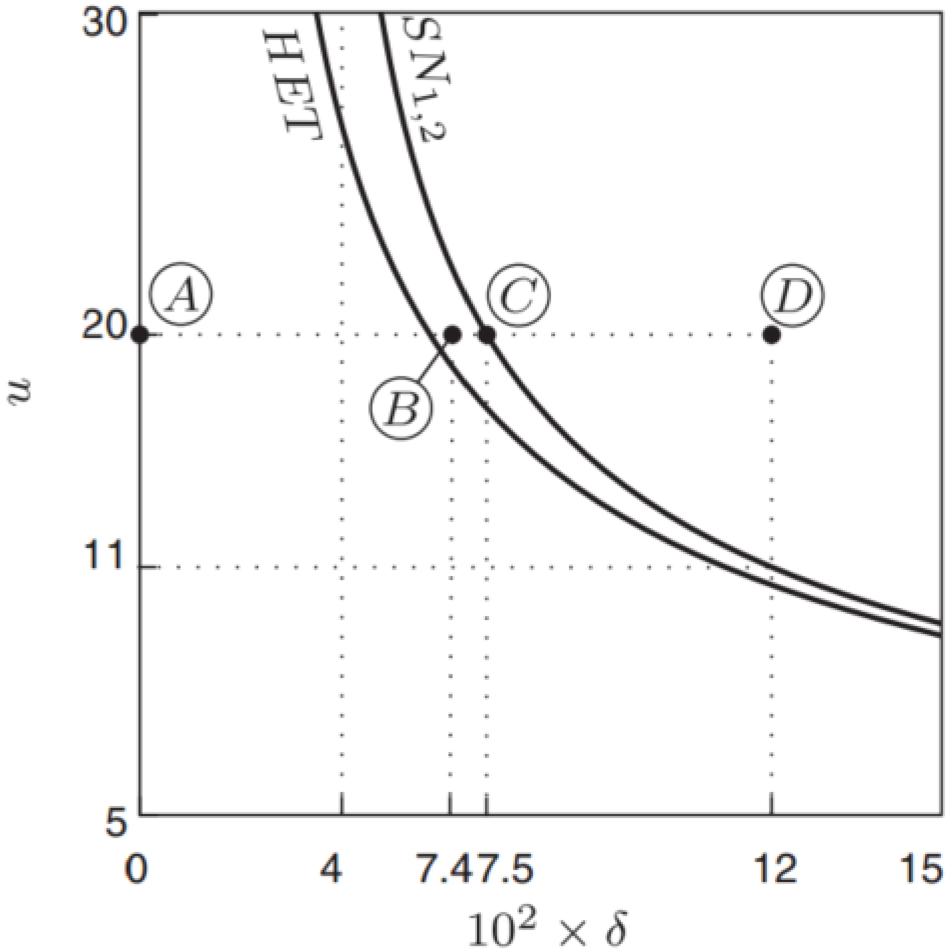
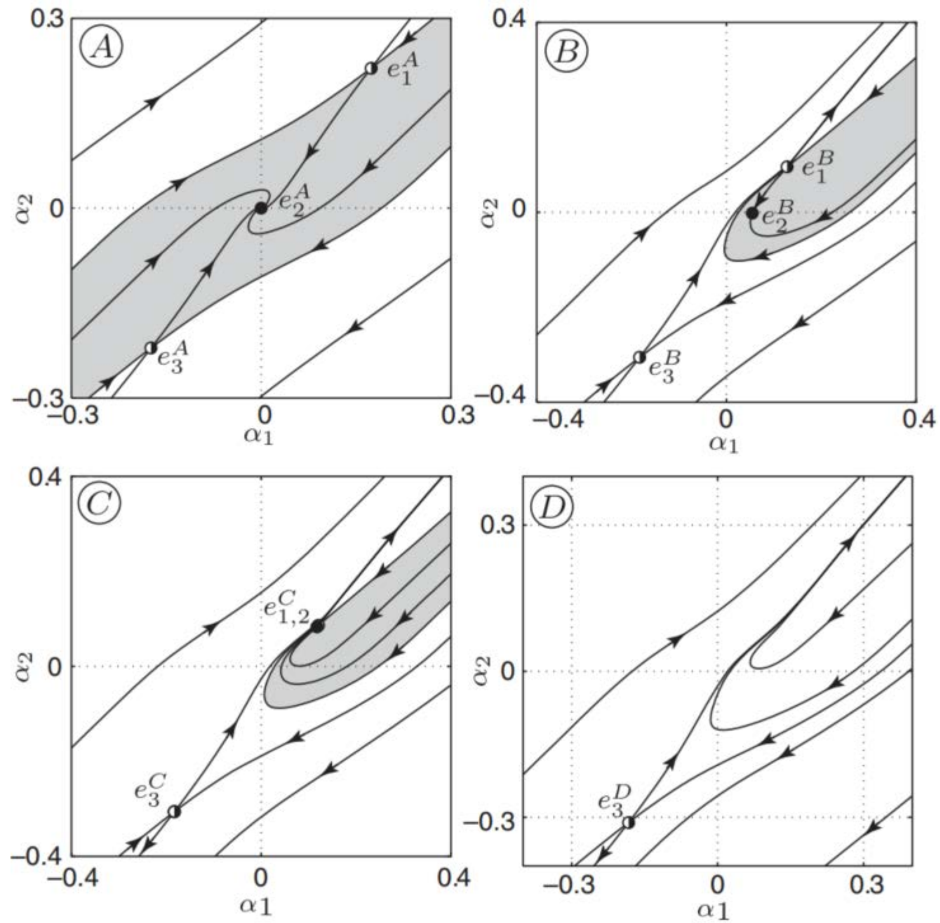
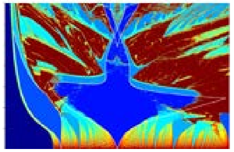
Bifurcation analysis– the UN_1 case





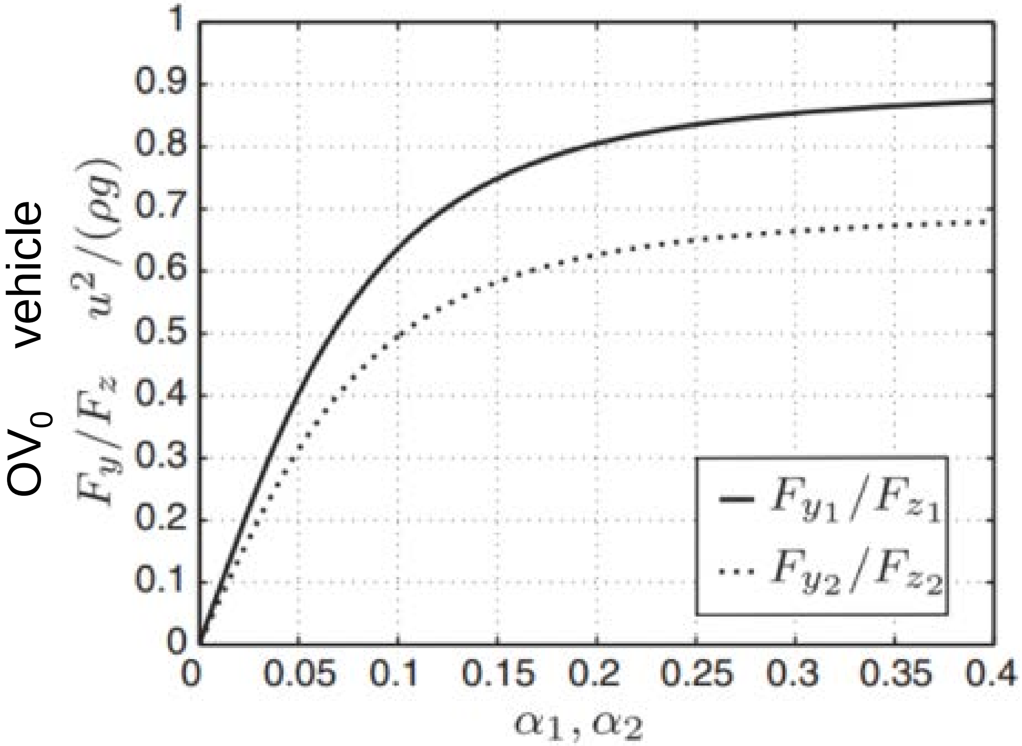


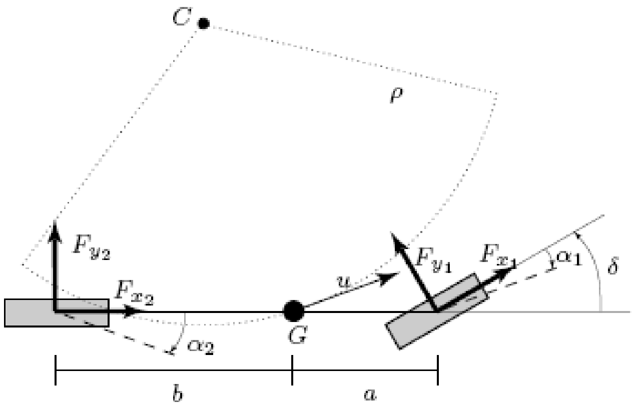


Rinaldi, Piccardi, Dercole, Gragnani, Miari, Colombo, Della Rossa

The single track model – non-linearities

Tyre characteristics

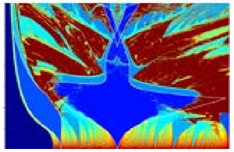




$$
\begin{aligned}
m(\dot{v} + ur) &= F_{y_1}(\alpha_1) + F_{y_2}(\alpha_2) \\
I_z \dot{r} &= F_{y_1}(\alpha_1)a - F_{y_2}(\alpha_2)b
\end{aligned}
$$

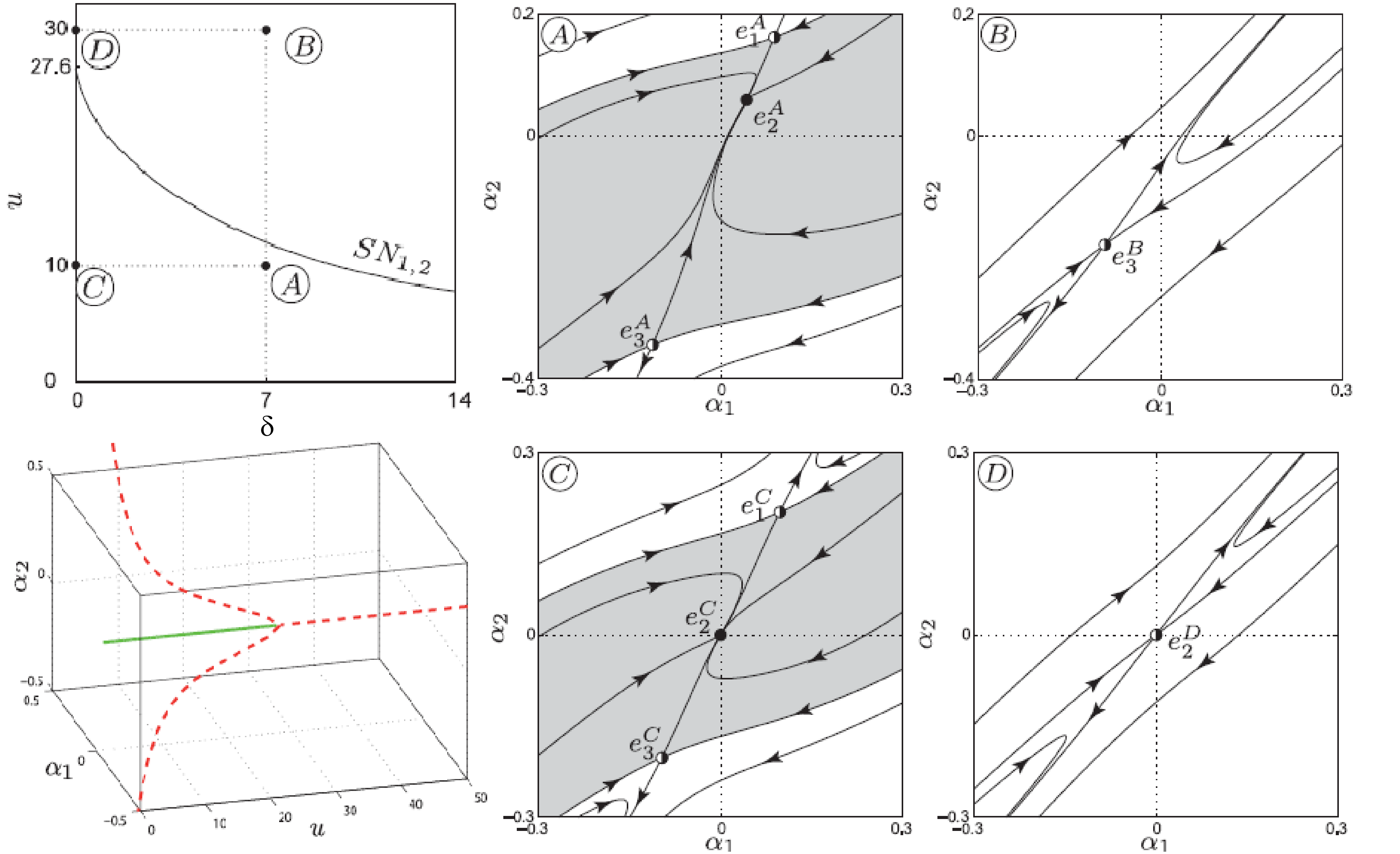
Where the rear and front slip angles can be obtained as:

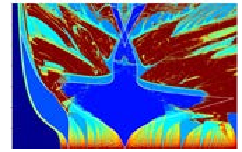
$$
\alpha_2 = -\frac{v - rb}{u}, \quad \delta - \alpha_1 = \frac{v + ra}{u}
$$



Try it with PPlane

Bifurcation analysis – OV_0 case





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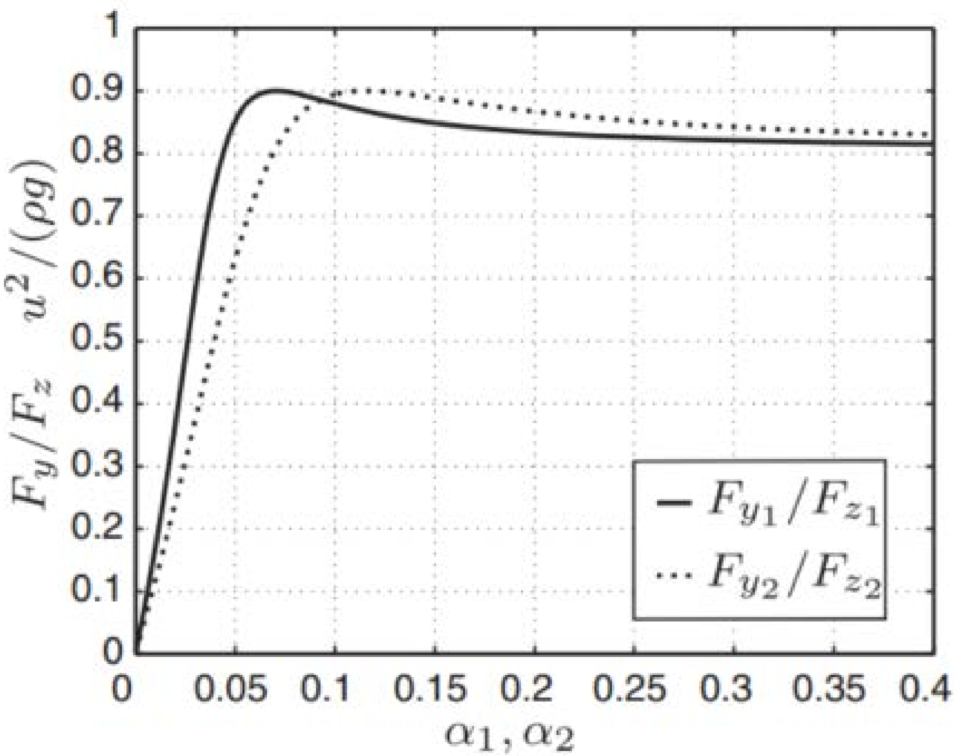
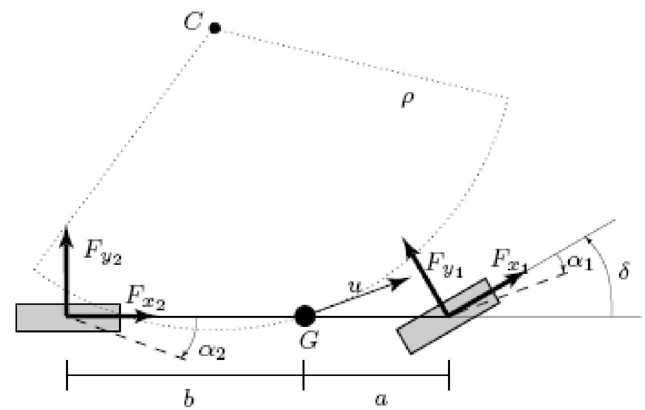
The single track model – non-linearities

Tyre characteristics

OV$_2$ vehicle

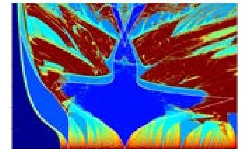




$$
\begin{aligned}
m(\dot{v} + ur) &= F_{y_1}(\alpha_1) + F_{y_2}(\alpha_2) \\
I_z\dot{r} &= F_{y_1}(\alpha_1)a - F_{y_2}(\alpha_2)b
\end{aligned}
$$

Where the rear and front slip angles can be obtained as:

$$
\alpha_2 = -\frac{v - rb}{u}, \quad \delta - \alpha_1 = \frac{v + ra}{u}
$$



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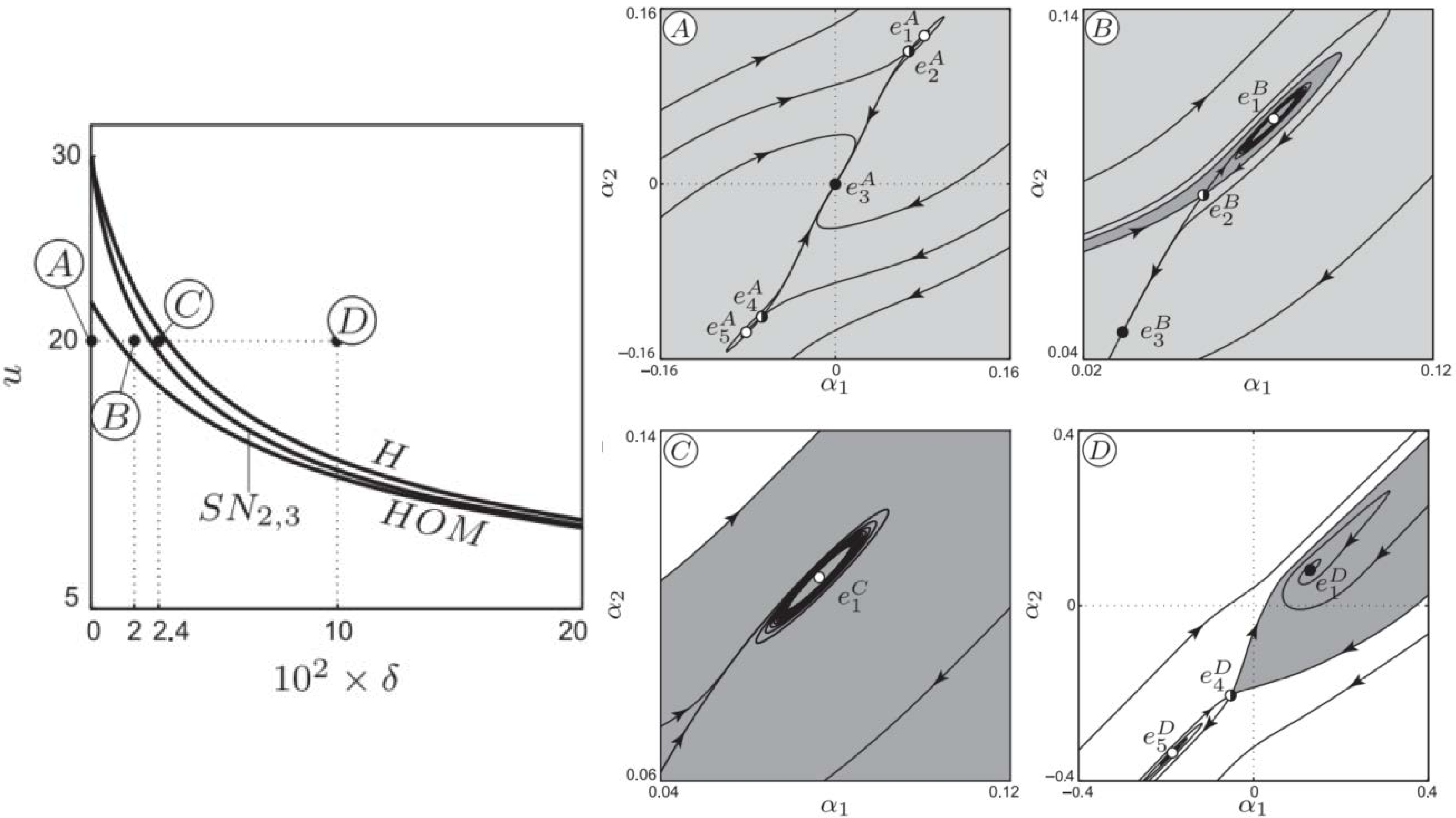
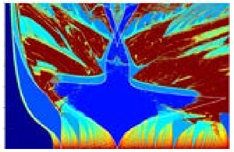
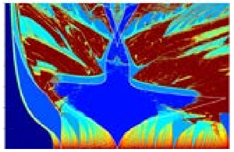
Bifurcation analysis – OV_2 case





Bifurcation analysis – some conclusions

- Propose a method that, for each car, shows in the parameter space the safety boundaries
- Find some behaviour that was already been seen in experiments but was never been founded in simulations
- Classify all the possible behaviours of this system and understand which are the causes that make vehicle unstable (or not robust)

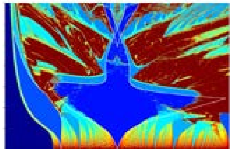


THE AUTOMOBILE MODEL AND ITS BA

- The single track model
- Model validation
- Bifurcation analysis of the vehicle model
- Understeering vs. Oversteering vehicles

THE HUMAN-IN-THE-LOOP SYSTEM

- Driver models and vehicle-driver coupling
- Model validation
- Bifurcation analysis of the vehicle and driver model
- Results comparison
- An extension and further research directions



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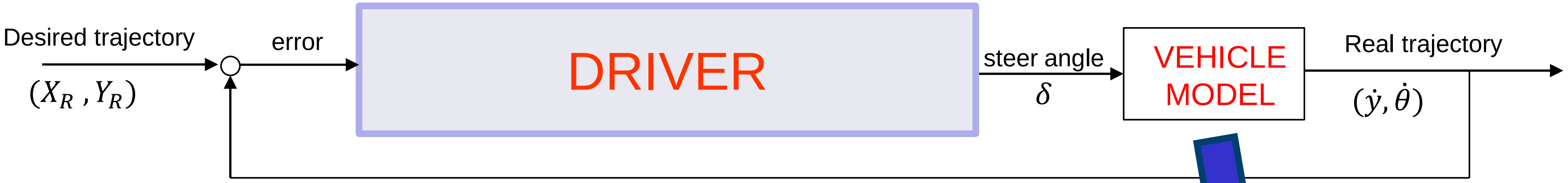
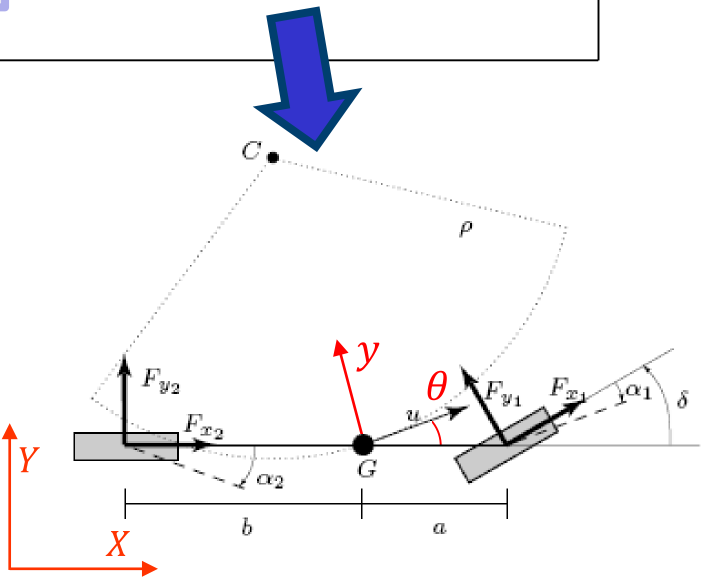
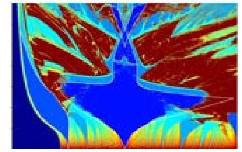
Vehicle+Driver model





SIMPLIFYING HYPOTHESES

- the forward speed *u* is constant;
- the centre of gravity lies at the ground level;
- the vehicle body is modelled referring to its longitudinal axis;
- the resultant of the forces acting at the front and rear axles are applied at the centres of the axles;
- the slip angles αi, $i = 1, 2$, and the steering angle δ (Figure 1) are small and
- no longitudinal forces are acting at the wheels.



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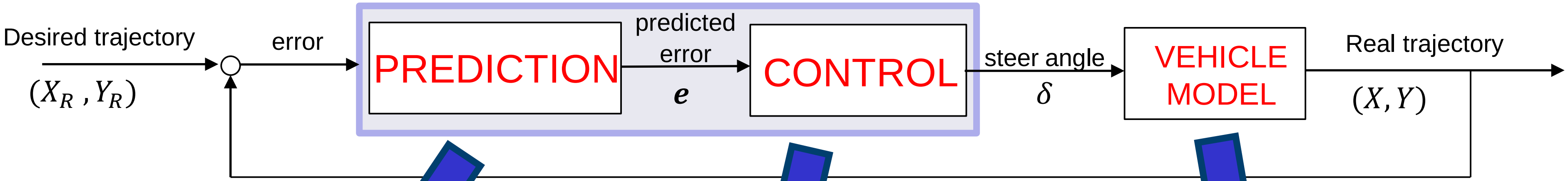
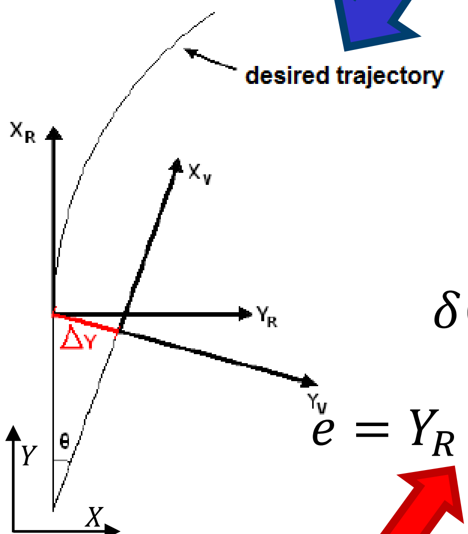
Vehicle+Driver model



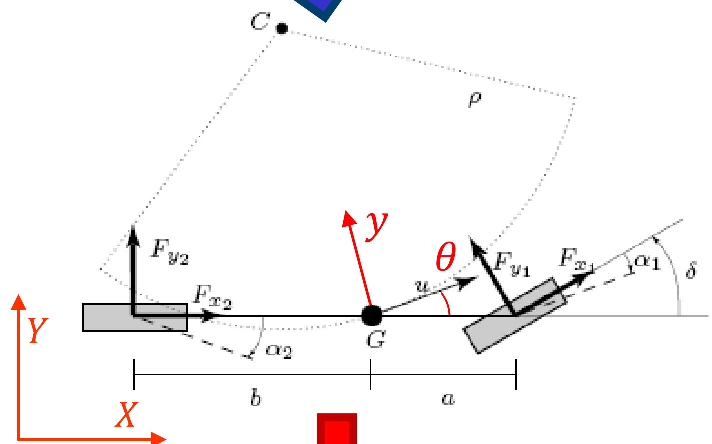


$$\delta(t+\tau_r) = h\,\boldsymbol{e}(t)$$

$$\delta(t) + \tau_r\dot{\delta}(t) = h\,\boldsymbol{e}(t)$$

$$e = Y_R - (Y + L\sin\theta)$$

$$\boldsymbol{e} = \begin{pmatrix} X_R \\ Y_R \end{pmatrix} - \left(\begin{pmatrix} X \\ Y \end{pmatrix} + L \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \right)$$



$$m\,(\ddot{Y} + u\,\dot{\theta}) = F_{y_1}(\alpha_1) + F_{y_2}(\alpha_2)$$

$$I_z\,\ddot{\theta} = F_{y_1}(\alpha_1)a - F_{y_2}(\alpha_2)b$$

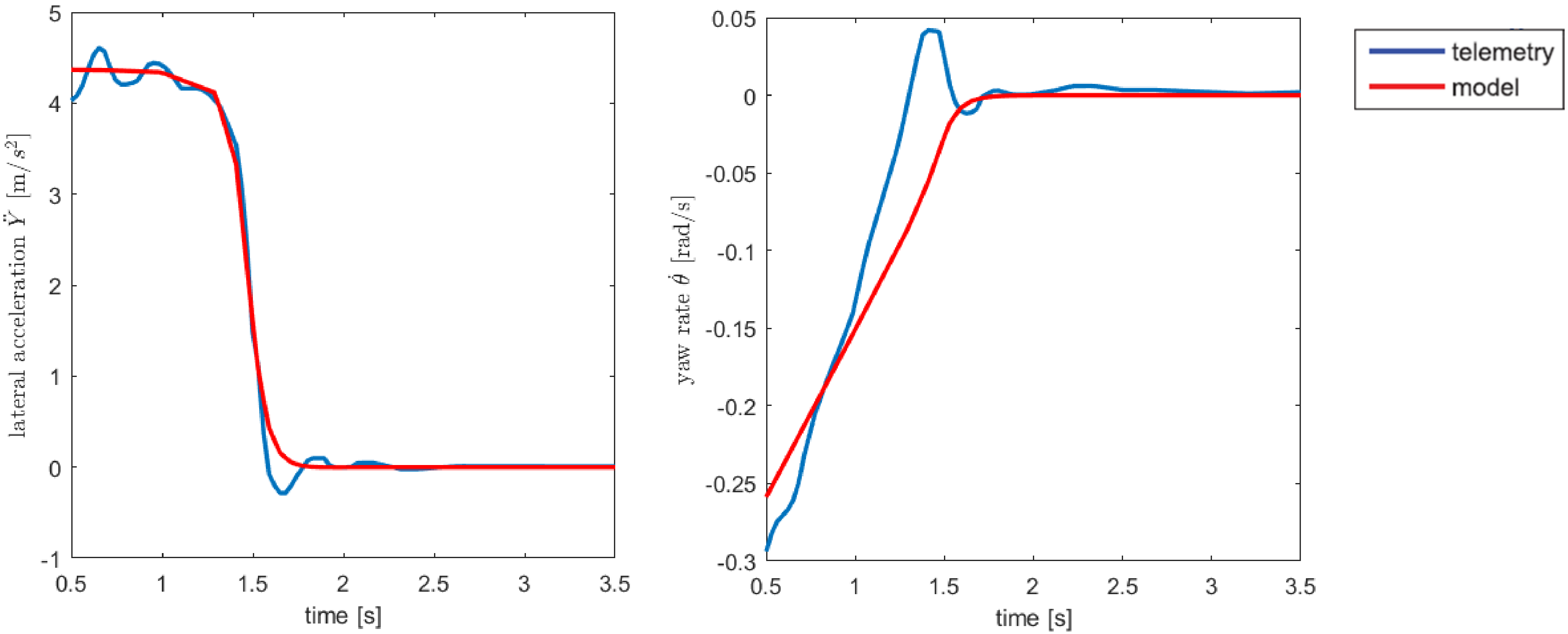
$$\alpha_2 = \theta - \frac{\dot{Y} - b\dot{\theta}}{u}, \qquad \theta + \delta - \alpha_1 = \frac{\dot{Y} + a\,\dot{\theta}}{u}$$

SIMPLIFYING HYPOTHESES

- …
- the desired trajectory is going straight at $Y_R = 0$;
- frequency of human steering < 3 Hz;

5 D.O.F. MODEL IN $(Y, \dot{Y}, \theta, \dot{\theta}, \delta)$

Model validation – Kick plate



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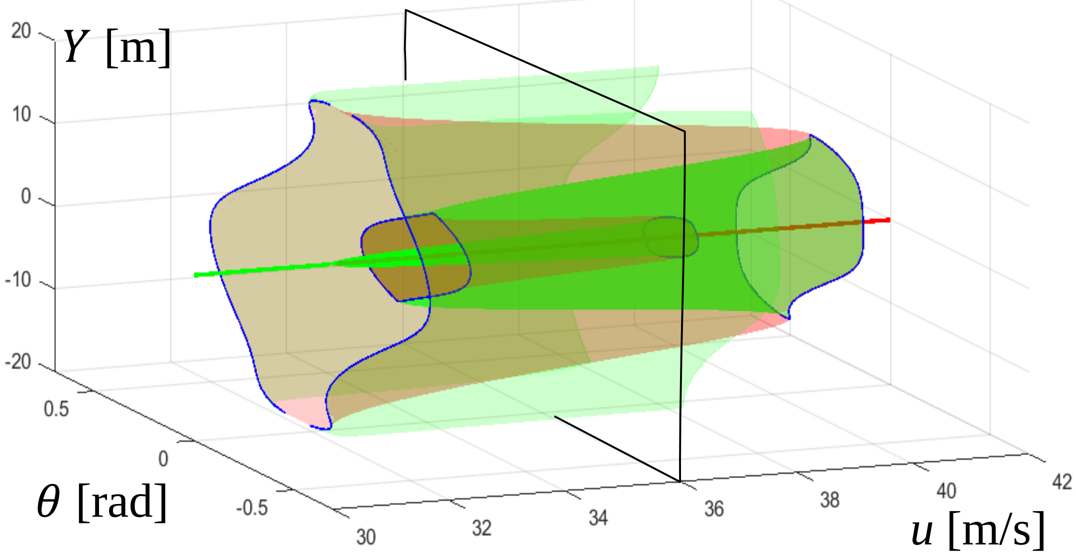
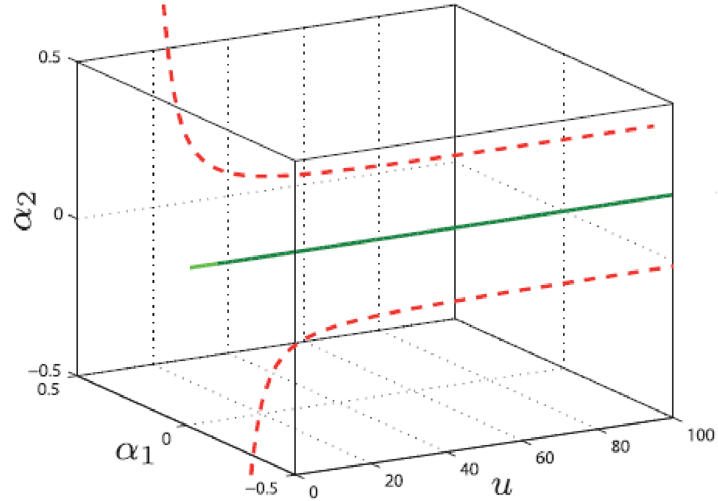
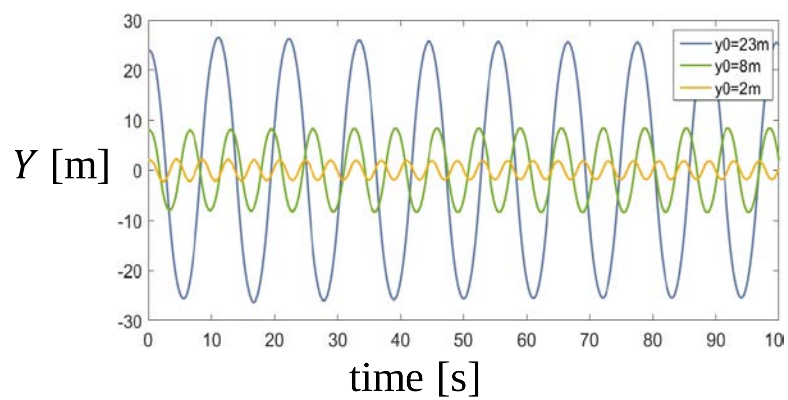
Miari, Colombo, Della Rossa

Result comparison: moving straight ahead at different speed

UN case



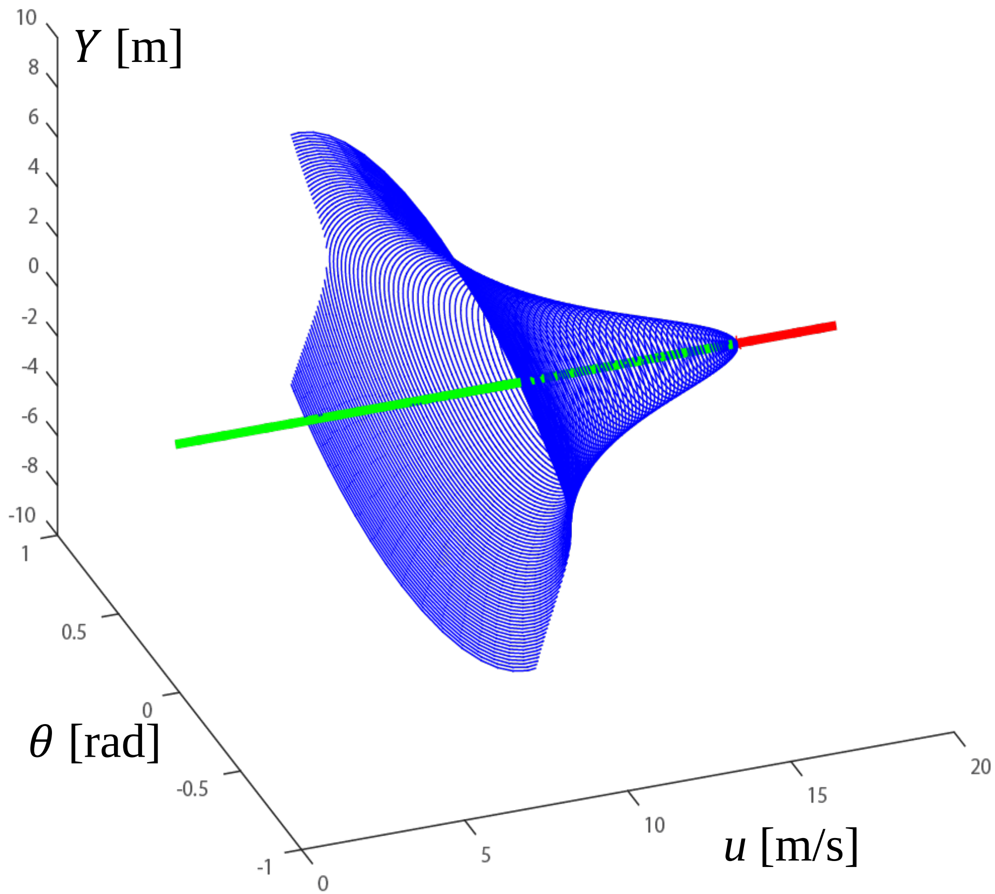


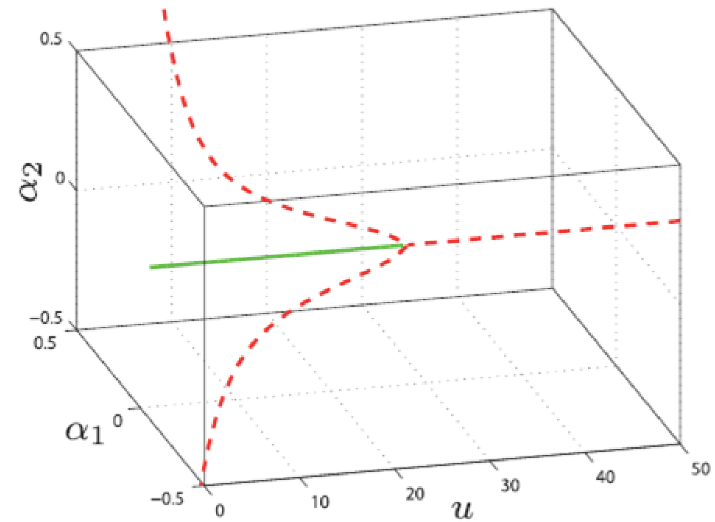


- Supercritical Hopf bifurcation
- Multiple tangent of limit cycle bifurcation

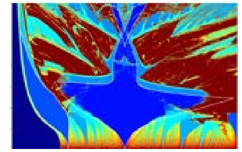
Result comparison: moving straight ahead at different speed

OV case

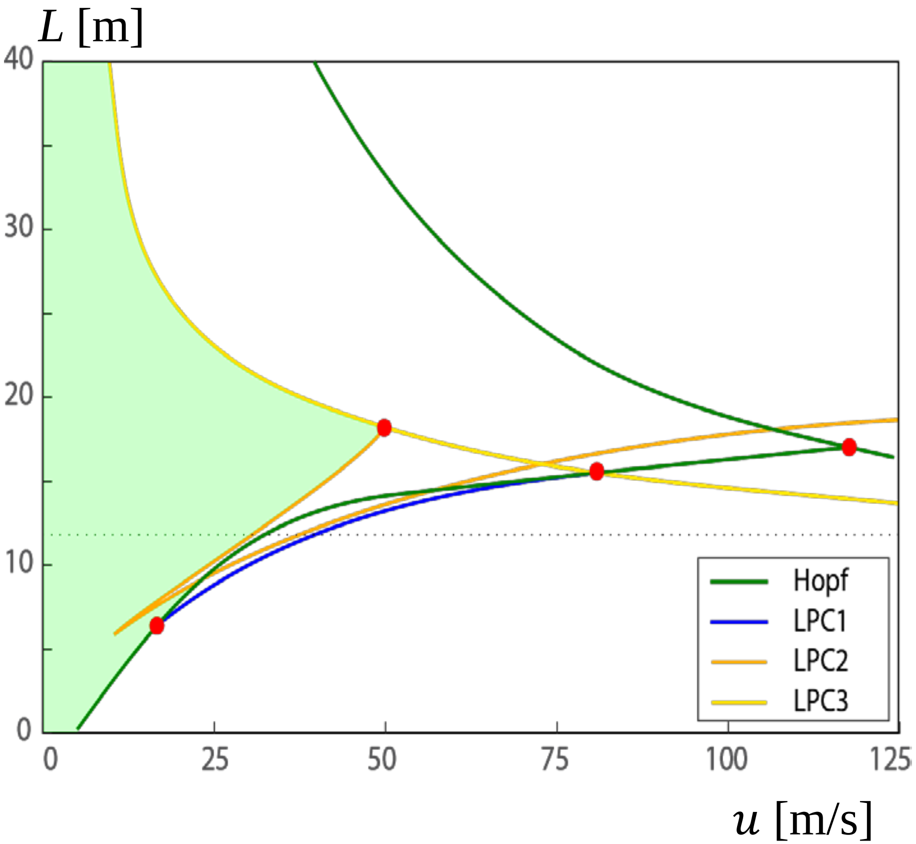


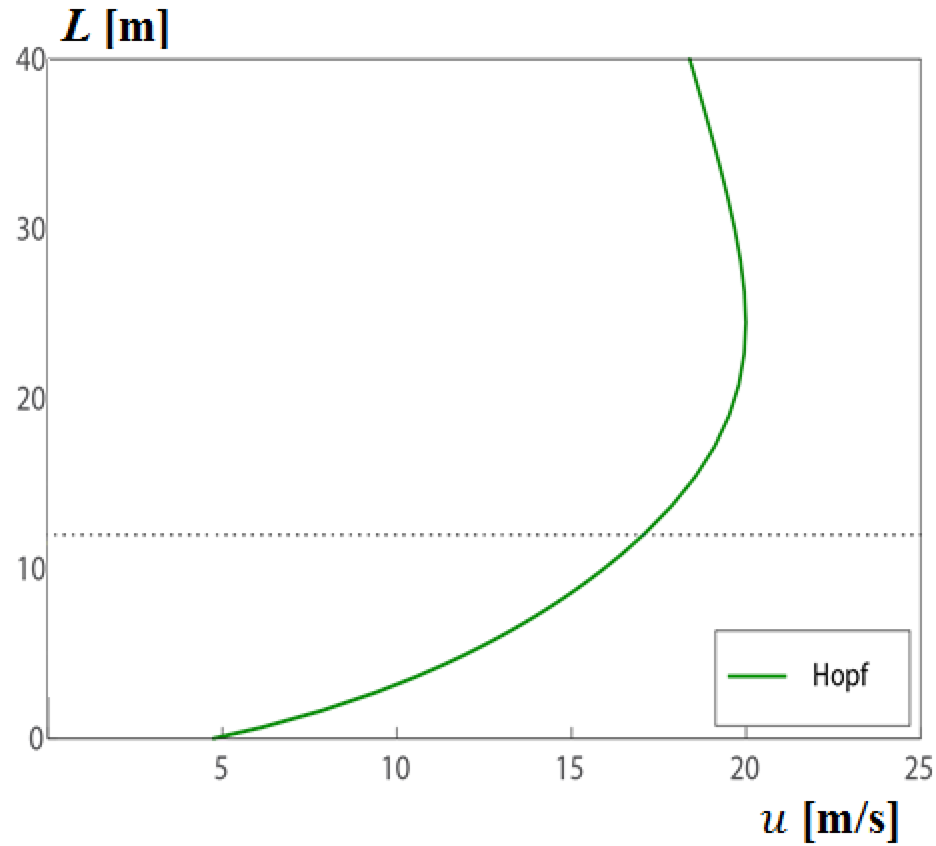


- Subcritical Hopf bifurcation

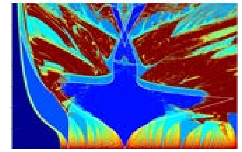


Comparison between UN and OV vehicles





L: length of prevision (driver ability in prevision)
u: forward velocity



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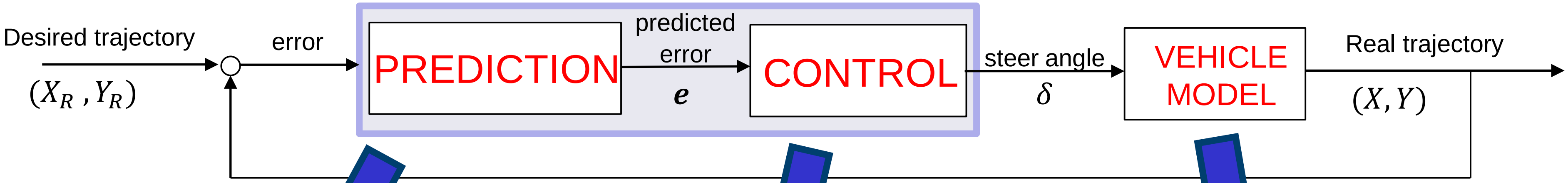
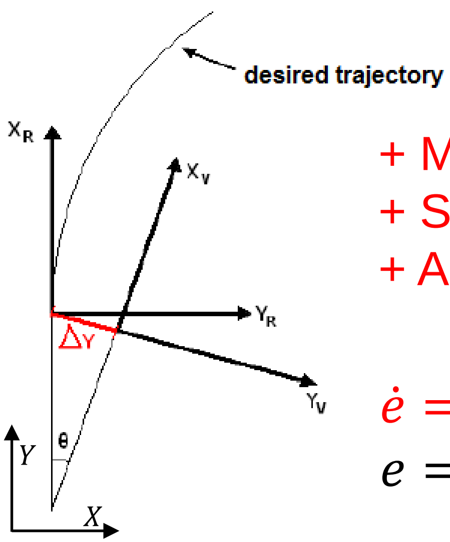


$$\delta(t+\tau_r) = h\,\boldsymbol{e}(t) + k\,\dot{\boldsymbol{e}}$$

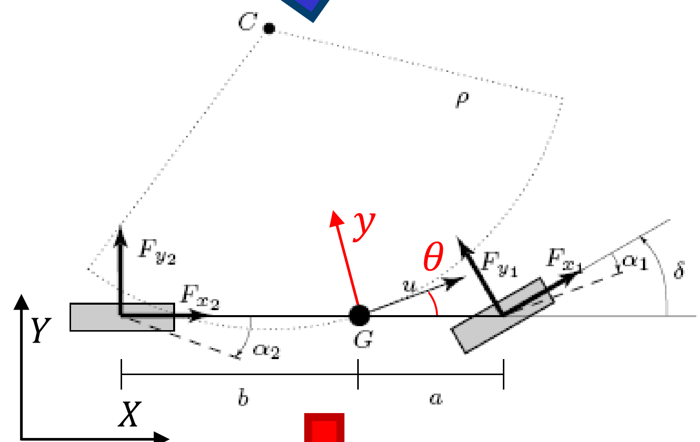


+ More human-like response
+ Should increase stability
+ Add freedom in control design

$$\dot{e} = -\dot{Y} - L\,\dot{\theta}\cos\theta$$

$$e = Y_R - (Y + L\sin\theta)$$

$$\boldsymbol{e} = \begin{pmatrix} X_R \\ Y_R \end{pmatrix} - \left(\begin{pmatrix} X \\ Y \end{pmatrix} + L \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \right)$$



$$m\,(\ddot{Y} + u\,\dot{\theta}) = F_{y_1}(\alpha_1) + F_{y_2}(\alpha_2)$$

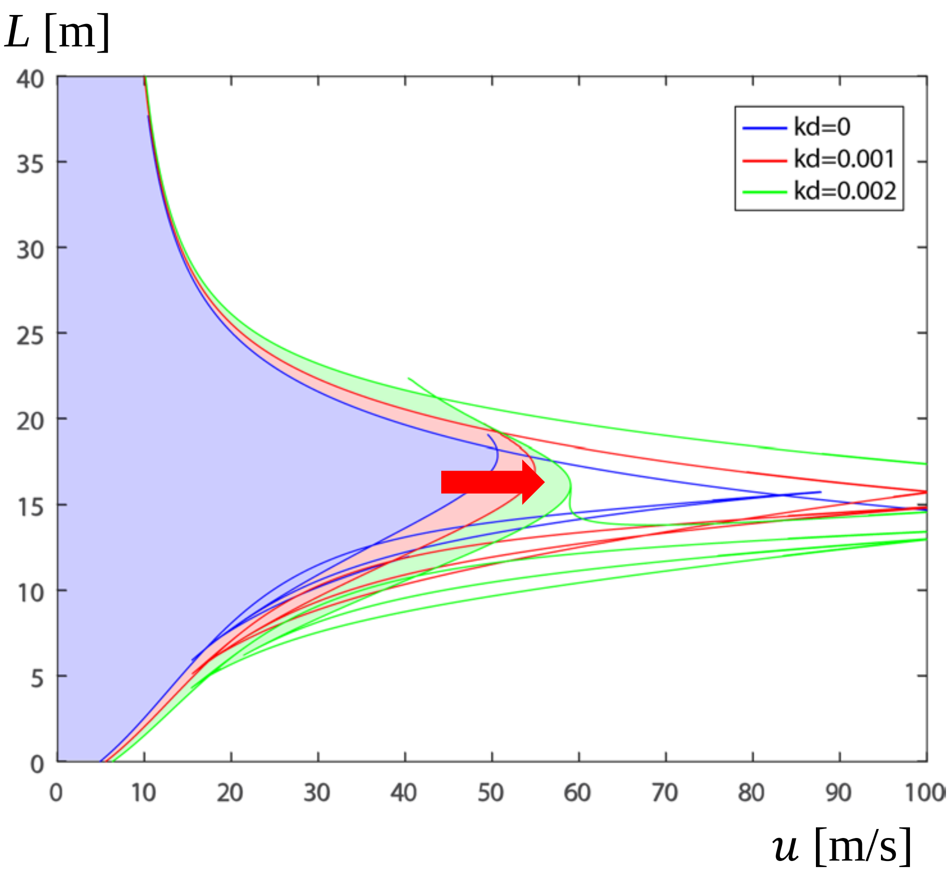
$$I_z\,\ddot{\theta} = F_{y_1}(\alpha_1)a - F_{y_2}(\alpha_2)b$$

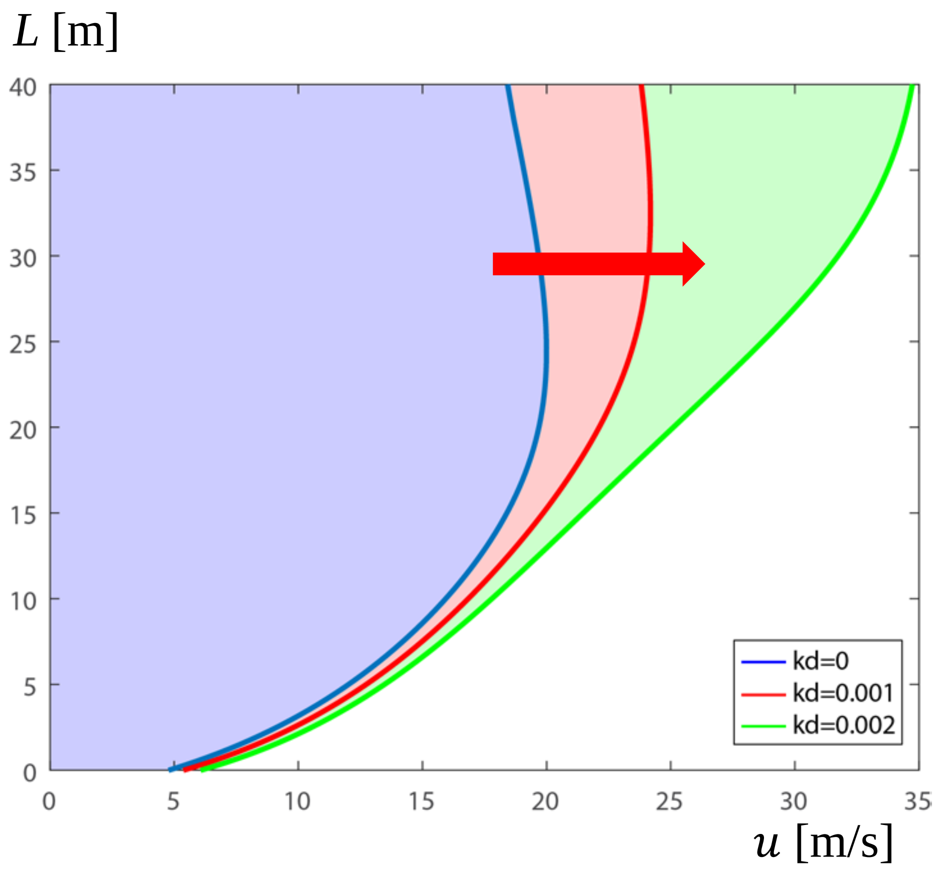
$$\alpha_2 = \theta - \frac{\dot{Y} - b\dot{\theta}}{u}, \qquad \theta + \delta - \alpha_1 = \frac{\dot{Y} + a\,\dot{\theta}}{u}$$

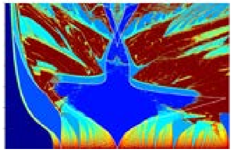
5 D.O.F. MODEL IN $(Y, \dot{Y}, \theta, \dot{\theta}, \delta)$

A more human-like control law take into account also the velocity with which the error changes!

$$\delta(t+\tau_r) = h\,\boldsymbol{e}(t) + k_d\,\dot{e}(t)$$







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Conclusions and further extensions (master thesis proposal ☺)

- Make the analysis of the vehicle+driver model for different running conditions (steering pad experiment)
- Quantify (in terms of distance from the nearest bifurcation curve) the robustness of a driving style, hence quantifying the ability of a driver
- Use bifurcation analysis in experiments to make a simple and direct comparison between different type of vehicles and different type of drivers