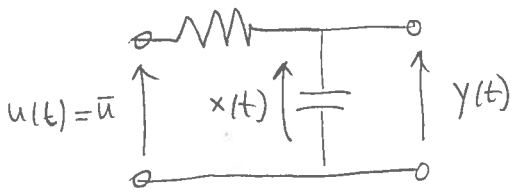


# time dependency

$$R(t) = R_0(1 + \varepsilon \sin \omega t) \quad \omega = \frac{2\pi}{T}, \quad T = 1 \text{ day}$$



$$\dot{x} = \frac{1}{C} \frac{u-x}{R}, \quad y = x \quad (\text{low-pass filter})$$

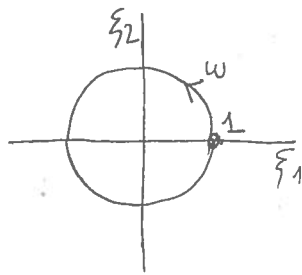
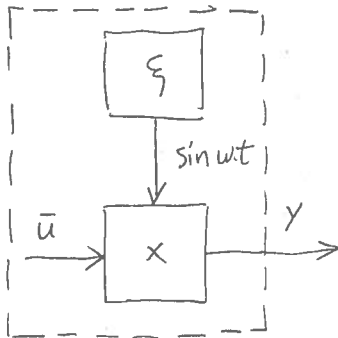
$$G(s) = \frac{1}{1+s\tau}, \quad \tau = RC : \text{time-constant}$$

$$\dot{x} = \frac{\bar{u} - x}{R_0 C (1 + \varepsilon \sin \omega t)} = f(x, t)$$

non-autonomous system

Note: constant inputs are considered together with model parameters and often not mentioned explicitly  $f(x, t, \bar{u}, p)$ ,  $p = (R_0, \varepsilon, \omega)$

autonomous extension

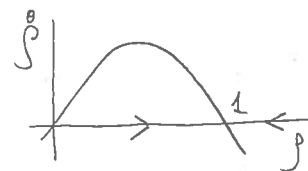


$$\xi_1(t) = \rho \cos \omega t$$

$$\xi_2(t) = \rho \sin \omega t$$

$$\dot{\rho} = \rho(1 - \rho^2), \quad \xi_1 = \rho \cos \vartheta$$

$$\dot{\vartheta} = \omega, \quad \xi_2 = \rho \sin \vartheta$$



$$\rho(t) \rightarrow 1 \quad t \rightarrow \infty$$

$$\vartheta(t) = \omega t$$

$$\dot{\xi}_1 = \dot{\rho} \cos \vartheta - \rho \omega \sin \vartheta$$

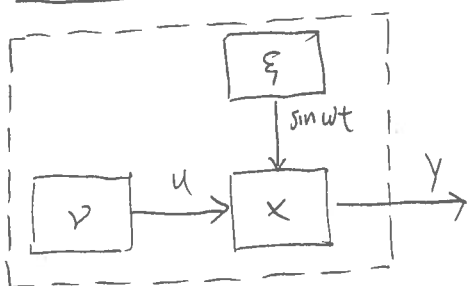
$$\dot{\xi}_2 = \dot{\rho} \sin \vartheta + \rho \omega \cos \vartheta$$

$$\dot{x} = \frac{\bar{u} - x}{R_0 C (1 + \varepsilon \xi_2)} = f(x, \xi_2)$$

$$\dot{\xi}_1 = \xi_1 - \omega \xi_2 - (\xi_1^2 + \xi_2^2) \xi_1 = h_1(\xi_1, \xi_2)$$

$$\dot{\xi}_2 = \omega \xi_1 + \xi_2 - (\xi_1^2 + \xi_2^2) \xi_2 = h_2(\xi_1, \xi_2)$$

non autonomous case



$$\text{state } z = \begin{bmatrix} x \\ \xi \\ v \end{bmatrix}$$

$$\dot{x} = f(x, u, t)$$

$\Downarrow$

$$\dot{x} = f(x, \xi_2, v)$$

$$\dot{\xi} = h(\xi)$$

$$\dot{v} = q(v)$$