Introduction to numerical bifurcation analysis

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Analyze the prey-predator Rosenzweig-MacArthur model when a constant super-predator is present:

$$\begin{aligned}\dot{x}_1 &= rx_1(1-\frac{x_1}{K})-\frac{a_1x_1}{b_1+x_1}x_2\\ \dot{x}_2 &= e\frac{ax_1}{b+x_1}x_2-mx_2-\frac{a_2x_2}{b_2+x_2}x_3\end{aligned}$$

where x_1 , and x_2 are the resource and predator densities, $r=e=a_2=1$, $a_1=5/3$, $b_1=1/3$, $b_2=1/2$, $m=0.12$, and x_3 is the super-predator density.

Analyze the different behavior of the model with respect to the resource richness K and to the super-predator density x_3.

Specifically:

- Interpret the model and discuss its properties (e.g., meaning of variables, positivity of the model, ...).
- Analyze through simulation (done with MatCont of PPlane) the behavior of the model for fixed K, and increasing x_3 (suggested values: 0, 0.5, 0.57, 0.6, 0.67, 0.69, 0.7, 0.75).
- Generate a bifurcation diagram in the space (x_3, x_1, x_2) in a rich environment ($K=1$) and in a poor environment ($K=0.2$).
- Generate a two-parameter bifurcation diagram for $(x_3, K)=[0,1]\times[0,2]$.