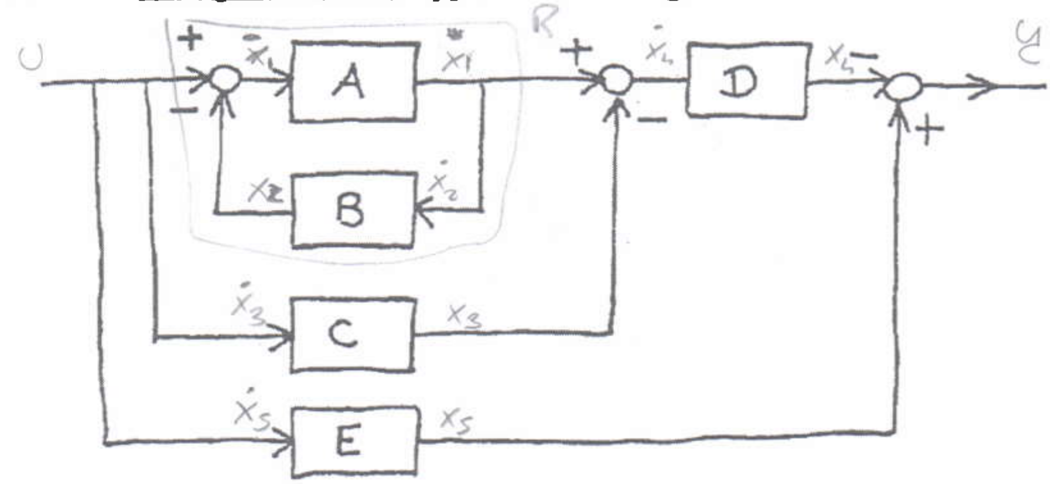
Esercizio 1

Si consideri il sistema *a tempo continuo* rappresentato in figura.



Si calcoli la funzione di trasferimento complessiva del sistema.
Supponendo che la funzione di trasferimento di tutti i sistemi sia $1/s$, si proponga una possibile realizzazione in spazio di stato del sistema.

1

$$\dot{x}_1 = u - x_2$$
$$\dot{x}_2 = x_1$$
$$\dot{x}_3 = u$$
$$\dot{x}_4 = x_1 - x_3$$
$$\dot{x}_5 = u$$
$$y = x_5 - x_4$$

$$G(s) = -\left(\frac{A}{1+AB} - C\right) D + E$$
$$= -\frac{1}{s}\left(\frac{1/s}{1+1/s^2} - \frac{1}{s}\right) + \frac{1}{s} = -\frac{1}{s}\left(\frac{s}{s^2+1} - \frac{1}{s}\right) + \frac{1}{s} =$$
$$= \frac{1}{s^2(s^2+1)} + \frac{1}{s} = \frac{s(s^2+1)+1}{s^2(s^2+1)}$$

$$s x_1 = u - x_2$$
$$s^2 x_2 = u - x_2 \Rightarrow (s^2+1) x_2 = u$$

$$\hookrightarrow s y = u - (x_1 - x_3)$$
$$s^2 y = su - (\cancel{u} - x_2) \cancel{+u}$$
$$s^2(s^2+1) y = (s(s^2-1)+1) u$$

ARRA DI ORDINE 4. PERCHE?

$$(s^n + \alpha_1 s^{n-1} + \dots + \alpha_n) y = (\beta_1 s^{n-1} + \dots + \beta_n) u$$

FORMA CANONICA DI RICOSTRUZIONE

$$A_r = \begin{bmatrix} 0 & 0 & \dots & 0 & -\alpha_n \\ 1 & 0 & \dots & 0 & -\alpha_{n-1} \\ 0 & 1 & \dots & 0 & -\alpha_{n-2} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -\alpha_1 \end{bmatrix} \quad B = \begin{bmatrix} \beta_n \\ \vdots \\ \beta_1 \end{bmatrix}$$

$$C^T = [0 \; \dots \; 1] \quad D = \beta_0$$

$$A_r = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$C^T = [0 \; 0 \; 0 \; 1] \quad D = 0$$

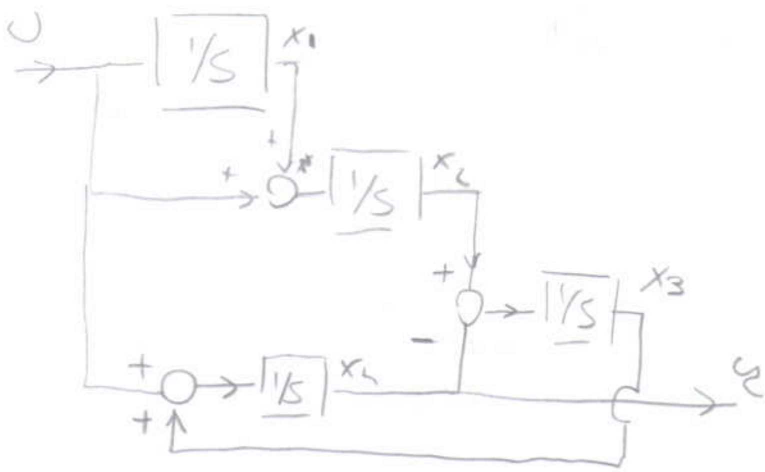
$\dot{x}_1 = u$

$\dot{x}_2 = x_1 + u$

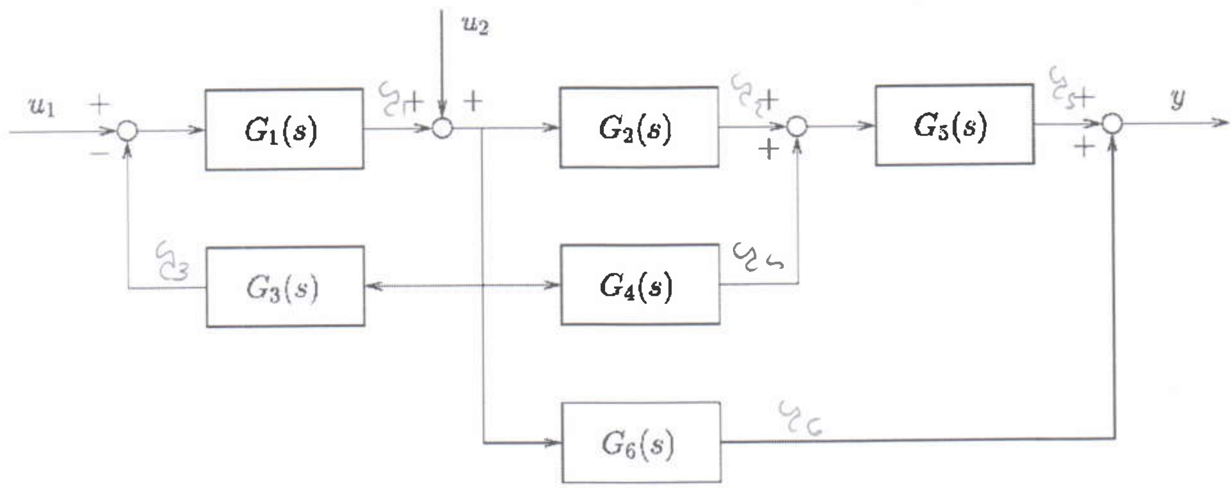
$\dot{x}_3 = x_2 - x_3$

$\dot{x}_4 = x_3 + u$

$y = x_4$

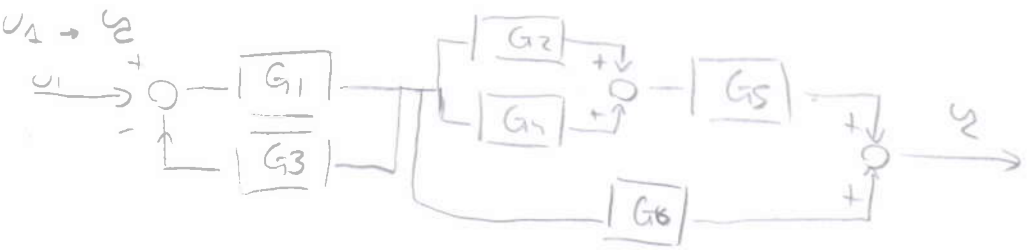


Esercizio 2

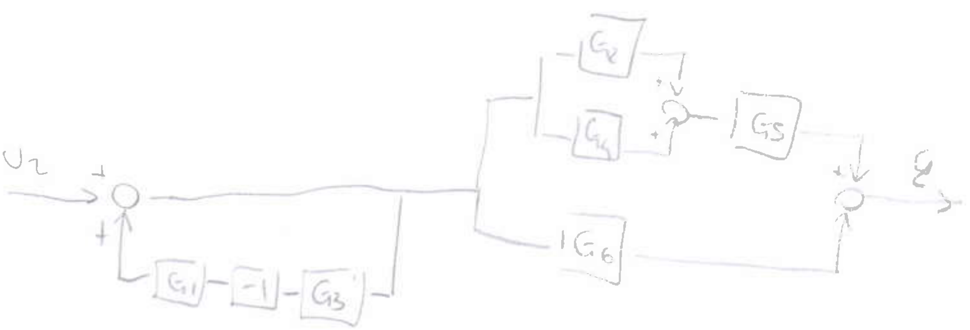


Si calcoli la funzione di trasferimento complessiva del sistema.

$u_1 \to y$



$$\frac{G_1}{1+G_1G_3}\left(G_6+\left(G_2+G_4\right)G_5\right)$$



$$\frac{1}{1+G_1G_3}\left(\left(G_2+G_4\right)G_5+G_6\right)$$

$y = y_5 + y_6$

$y_6 = (y_1 + u_2)\, G_6$

$y_5 = (y_2 + y_4)\, G_5$

$y_2 = G_2\,(u_2 + y_1)$

$y_4 = G_4\,(u_2 + y_1)$

$$\begin{cases} y_1 = G_1\,(u_1 - y_3) \\ y_3 = G_3\,(u_2 + y_1) \end{cases} \Rightarrow y_3 = G_3\left(u_2 + G_1(u_1 - y_3)\right)$$

$y_3(1+G_1G_3) = G_3 u_2 + G_1G_3 u_1$

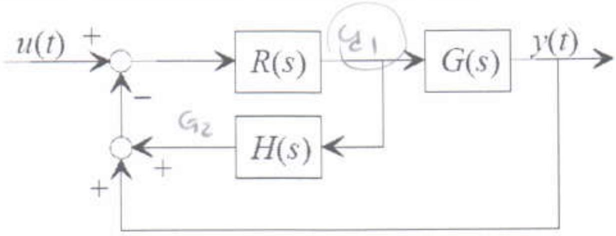
$y_1 = \frac{G_1}{1+G_1G_3}\left((1+G_1G_3)u_1 - G_3u_2 - G_1G_3u_1\right)$

...

$$u_2 + y_1 = \frac{G_1}{1+G_1G_3}\,u_1 + \frac{1+\cancel{G_1G_3}-\cancel{G_1G_3}}{1+G_1G_3}\,u_2$$

Esercizio 3

Si consideri il sistema descritto dal seguente schema a blocchi:



dove: $R(s) = k$, $H(s) = \dfrac{1}{1+5s}$, $G(s) = \dfrac{1}{1+10s}$

1.1) Determinare la funzione di trasferimento $F(s)$ tra $u(t)$ e $y(t)$. Commentare il risultato ottenuto

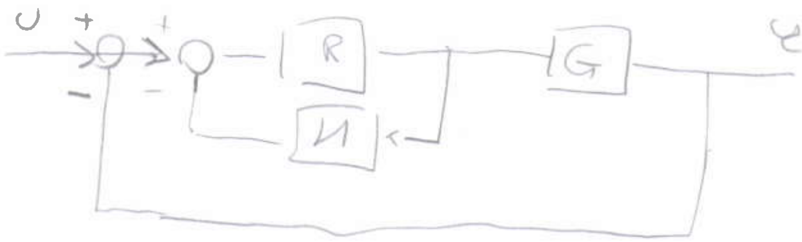
~~1.2) Determinare per quali valori del parametro k il sistema è asintoticamente stabile.~~

1.3) Denotando con $x_1(t)$ l'uscita del blocco $G(s)$ e con $x_2(t)$ l'uscita del blocco $H(s)$ e ponendo $k = 1$, determinare una rappresentazione in forma di stato del sistema.

1.4) Proporre un'altra rappresentazione in forma di stato del sistema

1.1)

Proviamo a ridisegnarlo



$$\frac{\frac{RG}{1+RH}}{1+\frac{RG}{1+RH}} = \frac{RG}{1+RG+RH}$$

$$\Downarrow$$

$$\frac{\frac{k}{1+10s}}{1+\frac{k(1+5s)+(1+10s)}{(1+5s)(1+10s)}}$$

$$\Downarrow$$

$$\frac{k(1+5s)}{(1+5s)(1+10s)+k(2+15s)}$$

$$y = G e_1$$

$$e_1 = R(u - y - e_2)$$

$$e_2 = H e_1$$

$$e_1 = R(u - G e_1 - H e_1)$$

$$\Downarrow$$

$$e_1(1 + RG + RH) = Ru$$

$$(1+10s)\,x_1 = u - x_1 - x_2$$

$$(1+5s)\,x_2 = u - x_1 - x_2$$

$$y = x_1$$

$$\Rightarrow \begin{cases} 10\dot{x}_1 = -2x_1 - x_2 + u \\ 5\dot{x}_2 = -x_1 - 2x_2 + u \end{cases}$$

$$(2+10s)\,x_1 = u - x_2$$

$$(2+5s)(2+10s)\,x_1 = (2+5s)u - u + x_1$$

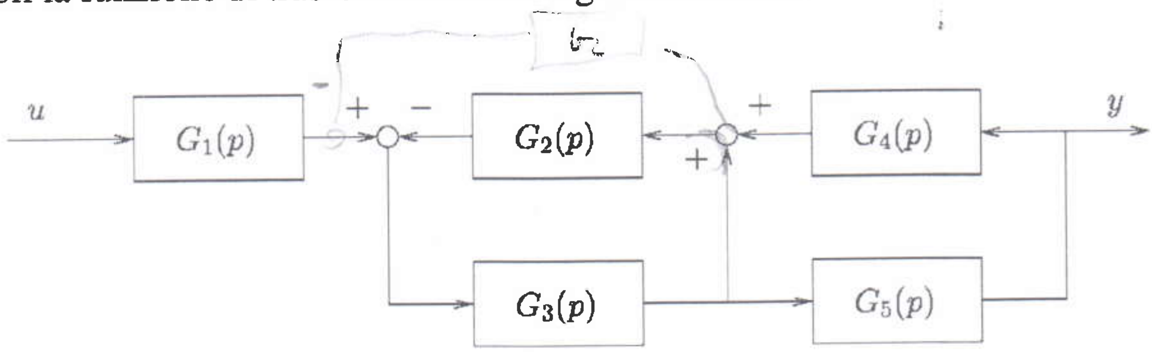
$$(50s^2+30s+3)\,y = (1+5s)u$$

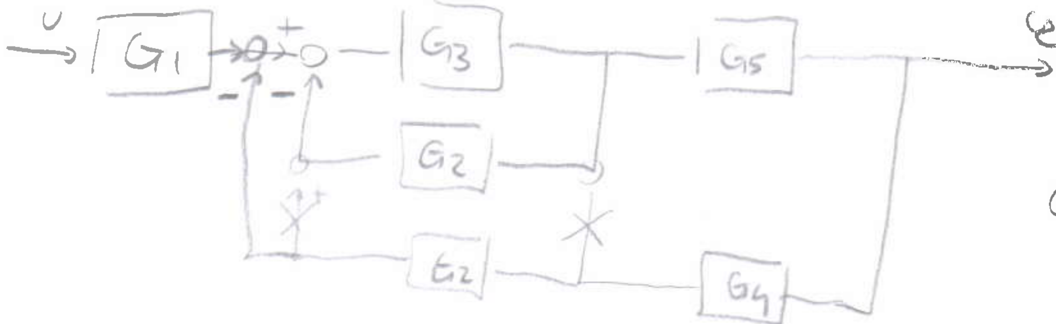
$$A_r = \begin{bmatrix} 0 & -3/50 \\ 1 & -3/5 \end{bmatrix} \quad B = \begin{bmatrix} 1/50 \\ 1/10 \end{bmatrix}$$

$$C = [0 \quad 1] \quad D = 0$$

Esercizio 4

Si calcoli la funzione di trasferimento del seguente sistema





$$G_1 \cdot \frac{\dfrac{G_3G_5}{1+G_2G_3}}{1+\dfrac{G_2G_3G_4G_5}{1+G_2G_3}} =$$

$$= \frac{G_1G_3G_5}{1+G_2G_3+G_2G_3G_4G_5}$$

$y = y_5$

$y_5 = G_5 y_3$

$y_3 = G_3(y_1 - y_2)$

$y_1 = G_1 u$

$y_2 = G_2(y_4 + y_3)$

$y_4 = G_4 y$

$$(1+G_2G_3+G_2G_3G_4G_5)\, y = G_1G_3G_5\, u$$

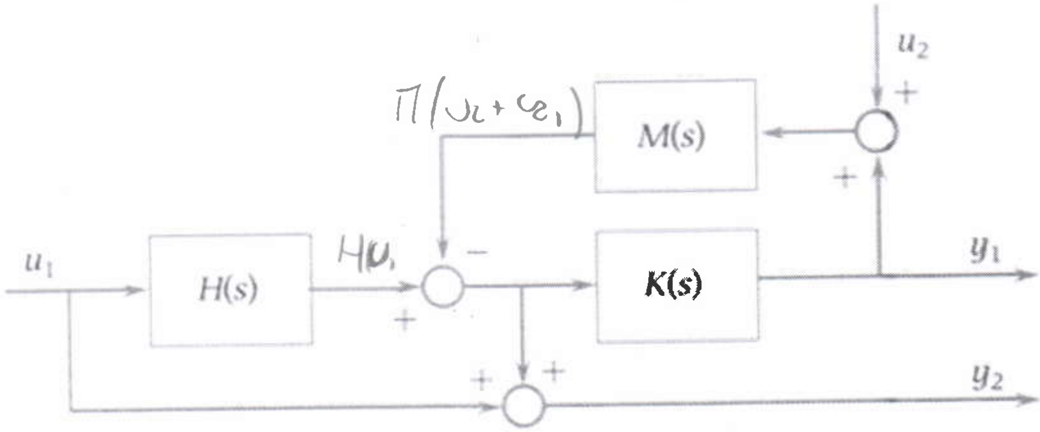
$$\Uparrow$$

$$y = y_5 = G_5 \cdot \frac{G_3}{1+G_2G_3}(G_1 u - G_2G_4 y)$$

$$y_3 = G_3(G_1 u - G_2G_4 y - G_2 y_3)$$

Esercizio 5

Si calcolino le funzioni di trasferimento del seguente sistema



- $y_1 = K\left(H u_1 - M(u_2 + y_1)\right) \Rightarrow y_1 = \frac{HK}{1+MK} u_1 - \frac{MK}{1+KM} u_2$

- $y_2 = u_1 + H u_1 - M(u_2 + y_1) = \frac{1 + MK + H + \cancel{HKM} - \cancel{HKM}}{1 + MK} u_1$

$$- \frac{M(1 + \cancel{MK} - \cancel{MK})}{1 + MK} u_2$$